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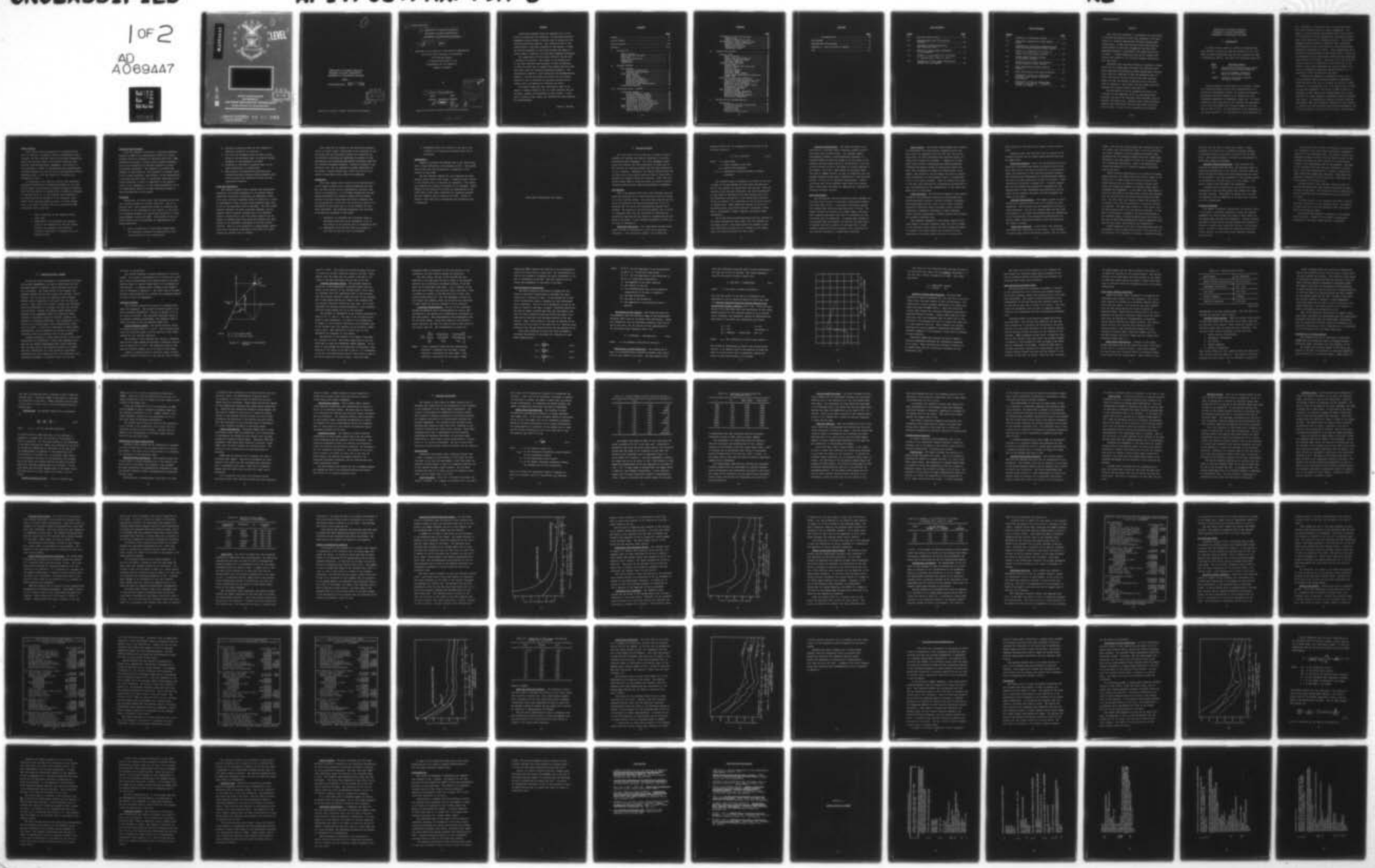

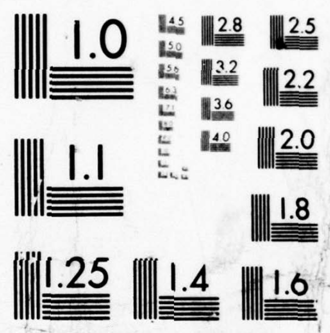

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Wright-Patterson Air Force Base, Ohio

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SENSITIVITY OF AIRCRAFT ATTRITION
ESTIMATES TO AIMING DISTRIBUTION
PARAMETERS OF ANTIAIRCRAFT ARTILLERY

THESIS

AFIT/GST/MA/79M-6 Errol C. Wilkins
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14 AFIT/GST/MA/79M-6

6 # SENSITIVITY OF AIRCRAFT ATTRITION ESTIMATES TO AIMING DISTRIBUTION PARAMETERS OF ANTIAIRCRAFT ARTILLERY.

9 Masters THESIS,

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air Training Command
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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Preface

Sensitivity analysis plays an important role in the world of simulations and modelling. Antiaircraft artillery engagement models present a special type of problem in conducting sensitivity analysis. This problem is due to the random nature of the many variables in the system. I chose the problem addressed in this study because the nature of problem made it conducive to Monte Carlo sampling techniques.

The organization of this report reflects the way that I attacked the problem. The concept of the analysis was to use basic physical relationships, as much as possible, to build the model and to compute data points from the model that would allow the required sensitivity analysis. The Introduction, Chapter I, and Conclusions and Recommendations, Chapter V, outline the thrust and results of this thesis. Chapters II and IV describe the analysis methods and procedures used, and Chapter III describes the model.

My sincere thanks and deep appreciation must be extended to Captain George E. Orr, my thesis advisor, for his patience, support, and assistance in this effort. I also want to thank my wife, Rosie, and children for their patience and understanding.

Errol C. Wilkins

Contents

Contents

Contents

List of Tables

List of Figures

ABSTRACT

This study was conducted to investigate the sensitivity of aircraft probability of kill (P_k) estimates to the parameters of a supplied antiaircraft artillery aiming angle distribution. A model was developed using basic physical relationships for the shell propagation and Monte Carlo sampling techniques to incorporate the random errors into the model. A shaped target was used and all random error sources were assumed to be bivariate normally distributed with zero means.

Simulation runs were made holding all variables constant except for the aiming angle variances and response curves were constructed by plotting P_k versus aiming angle variance. Tests were also conducted to test the assumption that the shell distribution on the encounter plane was bivariate normal and that the effects of gravity drop on the shell could be neglected. The results from the study model were compared with simular results from model P001.

The sensitivity of the P_k estimates was found to be a function of the magnitude of the P_k, the aiming angle variance, the sum of the ballistic error variances, and the aircraft vulnerable area. Response curve trends indicated slightly more sensitivity from the study model than from P001, however, it could not be concluded with certainty.

SENSITIVITY OF AIRCRAFT ATTRITION ESTIMATES TO AIMING DISTRIBUTION PARAMETERS OF ANTIAIRCRAFT ARTILLERY

I Introduction

In recent years a number of studies and analyses have been conducted in which the expected attrition of U.S. aircraft by enemy antiaircraft guns has been evaluated by means of mathematical models. The three most prominent models are:

Model	Developing Agency
EVADE	Army Material Systems Analysis Agency Aberdeen Proving Ground, Maryland
POO1	Air Force Armament Laboratory Eglin Air Force Base, Florida
SIMFIND	Institute for Defense Analyses Arlington, Virginia

The main purpose of these models is to provide a method for evaluating optimal tactics and countermeasures to be employed by penetrating aircraft into hostile environments. Antiaircraft Artillery models can be divided into four functional components (or submodels), these are: (1) determination of the flight path of the aircraft, (2) determination of the lead azimuth and elevation angles of the gun, and the time of fire, (3) determination of the trajectory of the fired projectile, (4) calculation of the probability of

hit. Simulation of these four areas can take many forms, from purely abstract models to actual engagements. The degree of realism modeled will greatly affect the precision of the probability of hit estimates and operational usefulness of the model(cost of running the model). Antiaircraft engagement models attempt to trade-off model operational effectiveness and real world realism by making simplifying assumptions throughout the models. These assumptions, which decrease the sophistication of the models, tend to smooth out the variations in the system while increasing the operational effectiveness of the model. Smoothing of the real world variations results in a distortion of the sensitivity of the models to their input parameters and therefore makes it difficult to determine a particular parameter's effect on the probability of hit calculated by the model.

Currently the Analysis and Simulation Branch of the 6570th Aerospace Medical Research Laboratory (AMRL/MEB) at Wright-Patterson AFB, Ohio is studying the effects of human trackers in the man-machine loop of an antiaircraft artillery and aircraft engagement. The thrust of AMRL's study is to determine if some tactics and countermeasures make it more difficult for human trackers to adequately track an aircraft than others. To ascertain what factors influence tracking performance, AMRL has studied the different actions required of trackers and have developed sophisticated mathematical and hardware simulation models that will simulate these actions.

AMRL's Problem

While actual measured degradation in tracking performance is required to determine overall countermeasure effectiveness, the more important aspect is how these degradations affect the attrition rates to be expected during an antiaircraft artillery/penetrator engagement. This effect is being measured, at AMRL, by incorporating more sophisticated tracking submodels into the Air Force's Antiaircraft Artillery Simulation Model - P001, and using P001 to predict the probability of kill.

In preliminary work, AMRL has observed that apparently significant differences in tracking performance do not have as much impact upon attrition estimates as might be expected. While this may reflect an insensitivity of attrition estimates to tracking errors, it is also possible that mismatches in the sophistication and precision of the submodels, downstream of the tracking submodel, smooth out these differences. Two areas of concern have been identified by AMRL for further study, these are:

1. Lack of precision in the terminal effects model used,
2. Differences in the precision and sophistication of the submodels for the fire control director, gun dynamics, and ballistics encounter, especially in comparison with the tracking models.

Thesis Problem Statement

Since the extensive variations in tracking performance measured by AMRL do not significantly affect the overall aircraft attrition estimates of simulation model P001, AMRL sponsored this thesis to obtain a more accurate determination of the sensitivity of aircraft attrition to aiming distribution parameters. The problem to be studied and analyzed in this thesis is: How sensitive is aircraft attrition to aiming variances? Specifically, is the insensitivity of aircraft attrition rates to significant variations in the aiming parameters a result of mismatches in the sophistication of the submodels, or is the actual probability of hit not significantly affected by these variations because of other perturbations in the system?

Objectives

Intuitively, one would expect that tracking errors would have a definite impact on aircraft attrition. Thus it is expected that a more sophisticated method of trajectory and probability of hit determination will reflect, in the attrition estimates produced, the significant variations in tracking performance observed by AMRL. Verification of this expected result will be accomplished by completing the following objectives:

1. Given a distribution of gun barrel aiming angles and variances, determine the sensitivity of aircraft attrition to this distribution.

2. Calculate confidence bands for the estimates of attrition determined in objective 1.
3. P001 uses the assumption that the shell distribution on the encounter plane is bivariate normal; determine if this assumption is valid.
4. Determine if the affect of gravity drop, on the shell distribution, is significant.
5. Make specific recommendations concerning anti-aircraft artillery terminal effects submodels that would increase their sensitivity along with the operational tradeoffs.

Scope and Limitations

The results of AMRL's tests indicate that significant variations in tracking performance do not provide equally significant variations in attrition, as predicted by P001. If in fact, attrition estimates should be sensitive to tracking performance, there are three major submodels where some or all of the variations could be smoothed: the fire control director submodel, the shell trajectory submodel, and the terminal effects submodel (probability of hit submodel). Losses in sensitivity in the fire control director are the subject of another study, this study will address only that portion of the simulation downstream of the fire control director. That is, only variations in aiming angles, muzzle velocities, trajectory disturbances, and methods of representing the aircraft will be studied.

This study will be limited to the design and implementation of simple dynamic simulations of the trajectory of the projectile and the interaction between the projectile and the aircraft, collecting and analyzing the results of such simulations, and determining appropriate confidence bands for the attrition estimates produced. Heuristic outcomes of the simulation will also be addressed as deemed appropriate by the author. Actual methods used in P001 will not be discussed or used except where needed for comparison.

Assumptions

This study assumes that acquisition and identification of the target aircraft pose no problem whatsoever for the gun, and that the gun is not constrained by doctrine, logistics, or reliability. It is further assumed that the gun aiming distribution is bivariate normal and that perturbations in the gun aiming angle distribution submodel have been accounted for, so that the only additional errors to be considered by this study will be in the external and terminal ballistics. The following assumptions are also made to decrease the variables in the study:

1. Difference in vulnerability of aircraft parts to projectile hits will not be considered, a hit on any portion of the aircraft will be considered a kill.
2. Engagements involving more than one aircraft or more than one gun will not be considered.

3. Engagements where the location of the gun or the aircraft is not known with certainty will not be considered.

Organization

Chapter II provides the methods used in the study along with a brief description of the methods of P001. The methods used in P001 are only mentioned as a comparison to the methods of this study.

The next chapter, Chapter III, will describe the mathematical model used by this study. A complete source listing of the computer code is contained in Appendix A. The remaining chapters will discuss the results of the analysis. First, Chapter IV will discuss the results and applications of the model used in this study, simulation run printouts supporting this analysis will be found in the text. Second, in Chapter V this work will be summarized and conclusions will be presented.

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II Analysis Methods

The first phase of this study was to design a model to simulate the terminal and exterior ballistics of an anti-aircraft/aircraft engagement. The tacit assumption underlying this analysis was that sensitivity is being lost in P001 between the tracking submodel and the final probability of kill estimates. Therefore, the logical starting point was to first look at the methods which are used in model P001's exterior and terminal ballistics submodels. Model P001 was chosen because AMRL is using that model to predict attrition estimate variations when tracking performance is degraded.

P001 Methods

P001 is an expected value model that evaluates only the means and variances of the errors associated with the inputs to the fire control system. The model then computes the aim-point and calculates the resulting probability of hit by assuming that the errors are independently normally distributed. Two aspects of the modelling methods will be briefly considered here: they are the method of modelling the projectile trajectory and the method of representing the aircraft in the model. A more detailed description of P001 can be found in reference 4.

Projectile Trajectory. The relationship between slant range and time is approximated in P001 by an empirical expression. This expression assumes a flat trajectory

(gravity affects are not considered), and is given by the following equation:

$$R = V_m t \ /(1+at+bt^2) \qquad (2\text{-}1)$$

where: R = slant range

t = time of flight of the shell

V_m = muzzle velocity(speed)

a & b = empirically derived projectile slowdown constants

The encounter plane is assumed to be normal to the line of sight to the gun through the point where the mean projectile and aircraft are equidistant from the gun. Once the encounter plane has been determined, the shell distribution on this plane is described by the mean aiming point and a root-sum-square conbination of the aiming angle variances (which are assumed to be independent), and six other sources of random error. These additional sources of error are: processing error, gun jitter, ballistic dispersion, atmospheric disturbance, flight roughness, and muzzle speed uncertainty.

All of the error sources mentioned above are included in the model developed for this study, with the exception of processing errors. Processing errors are assumed to have been previously accounted for and included in the aiming angle distribution provided for this study.

Aircraft Representation. The target aircraft is represented as a diffused target bivariate normally distributed across the encounter plane. The diffused target approximation defines a three-dimensional Cartesian coordinate system in which the origin is at the center of the aircraft, and the Z-axis points in the direction of the projectile velocity vector at the moment of closest approach to the target. The X and Y axes define the encounter plane. This type of damage function is used in conjunction with the assumed bivariate normal shell distribution so that the probability of kill can be determined analytically. Vulnerable areas used in the diffused target model are obtained from a table where the areas are expressed as a function of relative intercept velocity.

Model Development

The Antiaircraft Artillery Simulation Model (AAASIM) was developed with the idea of using more sophisticated methods than those used in P001. A projectile trajectory submodel was designed that used point mass equations of motion. The target aircraft was modelled as a shaped target, and Monte Carlo sampling techniques were used to incorporate the random errors into the simulation. In making these changes it was realized that the computer run time for the model would be greater than P001; however, it was hoped that a corresponding trade-off would be realized in the sensitivity of the attrition estimates to aiming angle variances.

Model Concept. The concept behind AAASIM was to develop an antiaircraft artillery/aircraft engagement model that would model the engagement from the time that the gun was fired until the shell intercepted the encounter plane; or, either hit or missed the aircraft. A sample run of this model would consist of specifying the system independent variables and making a number of replications which used Monte Carlo techniques to sample the random errors in the system. The results of any one sample run would be either a shell distribution on the encounter plane or a probability of hit on the target aircraft. Thus, by varying the inputs to the model and observing the corresponding probabilities of hit, the sensitivity of the probabilities to the input variables could be determined.

Model Variables. The variables exogenous to the system consist of the parameters defining the aiming angle distribution(aximuth variance, elevation variance, covariance, mean aiming angle errors), time of fire, muzzle velocity, projectile trajectory random errors(gun jitter, ballistic dispersion, atmospheric disturbance, flight roughness, muzzle velocity uncertainty), aircraft position and velocity, and aircraft vulnerable area.

The variables endogenous to the system consist of the slant range to the mean intercept point, the time of flight of the projectile until impact, the mean azimuth and elevation angles, the actual azimuth and elevation for each replication, the actual muzzle velocity of the shell for

each replication, and the point of impact on the encounter plane.

Variables input into the model that are related to the statistics desired are the random number starting seed and the sample size.

Trajectory Submodel. One of the sub-objectives of this study was to determine what affect the gravity drop of the shell would have on the shell distribution on the encounter plane, and ultimately on the attrition estimates produced. This objective was accomplished by including gravity affects into the point mass equations of motion of the shell. These equations are described in Chapter III. A further enhancement of this submodel was obtained by accounting for the varying nature of the aerodynamic drag coefficient for the shell, the air density, and speed of sound throughout the flight of the shell.

Aircraft Representation. The target aircraft is represented by an ellipsoid whose size can be defined by input parameters. This ellipsoid has the capability of moving in three dimensions. The use of a shaped target did away with the necessity to calculate the projected area for probability of hit calculations, thus enabling the calculations to be made by simply counting the number of hits and dividing by the number of trials.

Monte Carlo Sampling. A mean theoretical intercept point(MTIP) was calculated for each sample. From the MTIP the mean aiming azimuth and elevation angles could be deter-

mined. Once the mean aiming angles were calculated, each of the random error sources(except muzzle velocity uncertainty and flight roughness) could be sampled using Monte Carlo techniques and the resulting random errors were added algebraically to the mean angles to arrive at the aiming angles for a particular replication. Each of the random error sources was assumed to be bivariate normally distributed with a zero mean.

Muzzle velocity uncertainties do not affect the aiming angles directly, but rather affect the time of flight of the shell and to a lesser extent the amount of gravity drop. AAASIM accounts for the affects of muzzle velocity uncertainty by keeping the aircraft or encounter plane moving throughout the engagement. A slow shell will take longer to reach the same point as the mean shell, during this extended time the aircraft will have moved further along the flight path. The slow shell will therefore impact the aircraft at a different point than the mean shell, or miss the aircraft all together. The same logic applies to a fast shell, only the miss distance will be in the opposite direction. If the model is being used to determine the shell distribution of the encounter plane, a moving encounter plane will result in the slow or fast shell impacting the plane at different point than the mean shell, thus altering the distribution.

Flight roughness is the error source associated with the uncertainty of the aircraft position due to air currents and the inability of the aircraft to maintain a constant velocity,

from the time the shell is fired until impact. Flight roughness is accounted for in AAASIM by randomly shifting the aircraft position from where it would be for the mean shell. This random shifting of location is accomplished by Monte Carlo Sampling techniques.

Encounter Plane Determination. When the shell distribution is desired from the model, the encounter plane is positioned perpendicular to the relative velocity vector and passes through the moving point mass representing the center of the aircraft. Once the orientation of the encounter plane is determined for the mean shell, it is not changed. The plane, however, moves throughout the engagement.

A slight error is induced into the results since the orientation of the encounter plane would change with changing shell velocity. This effect is assumed to be so small in comparison with the magnitude of the mean shell velocity as to be insignificant.

Analysis Procedures

The number of different combinations of variables values in this model is infinite; therefore, to make the analysis manageable all error sources were fixed in value except for the aiming angle variances and covariance. Aircraft velocity was fixed in value and the aircraft position was adjusted to provide a constant slant range from the gun to the intercept point. In order to facilitate the comparison between different samples, the same random number generator starting seed was used.

The procedure used to test the sensitivity of the attrition estimates to the aiming angle distribution parameters consisted of the following steps. The aircraft position was adjusted by using the time of fire so that the point of mean intercept was where the shell velocity vector was perpendicular to the longitudinal axis of the aircraft. The range to the intercept point was also kept constant for a given test. With these parameters set, a series of samples were taken varying the aiming angle variances. Each sample run produced a probabiltiy of hit along with a confidence band corresponding to the sample size.

Another series of test were made using the same procedures described above, only without considering the gravity drop of the shell. This series was used to determine the affect of gravity drop on probabilities of hit produced by the model.

The final series of test conducted were made to provide the shell distribution on the encounter plane. The purpose of these runs was to test the assumption that this distribution was bivariate normal.

This section has provided a discussion of the methods of the analysis. Chapter III is devoted to presenting a description of the model AAASIM and how it functions.

III Simulation Model - AAASIM

In the real world environment, of antiaircraft artillery and aircraft engagements, the laws of nature have the most pronounced affect on whether a given artillery shell will hit an aircraft. The vulnerability of the aircraft will determine whether the hit will disable the aircraft. Specifically, once the gun has been aimed and fired, natural affects cause a dispersion of the shell from its mean trajectory. These affects are random in nature, and therefore, induce an uncertainty as to whether the shell and aircraft will intercept. Besides the natural random affects, the dynamics of the engagement affect the ability of human trackers to accurately track the aircraft, causing a degradation in tracking performance. This degradation is reflected in the aiming angles, thus inducing human errors into the system. A fortuitous combination of these errors, uncertainties, and perturbations can produce a projectile trajectory thich will intersect that of the aircraft.

Assuming something is known about the way in which each random error is distributed, a Monte Carlo sampling can be made and the results superimposed upon a mean trajectory. Thus the basic goal of this simulation model is to mathematically characterize the total projectile trajectory, superimpose the known random errors on this trajectory, and dynamically propagate a projectile until a hit or miss

situation is encountered.

This chapter presents a concise discussion of the model AAASIM. The concept of presentation is to provide the reader with a basic understanding of the methods used by the model. Explicit detail is omitted except where it is necessary for clarification. A complete source listing of the computer code is contained in Appendix A. The first step in the mathematical characterization is to define the coordinate systems in which to model the engagement.

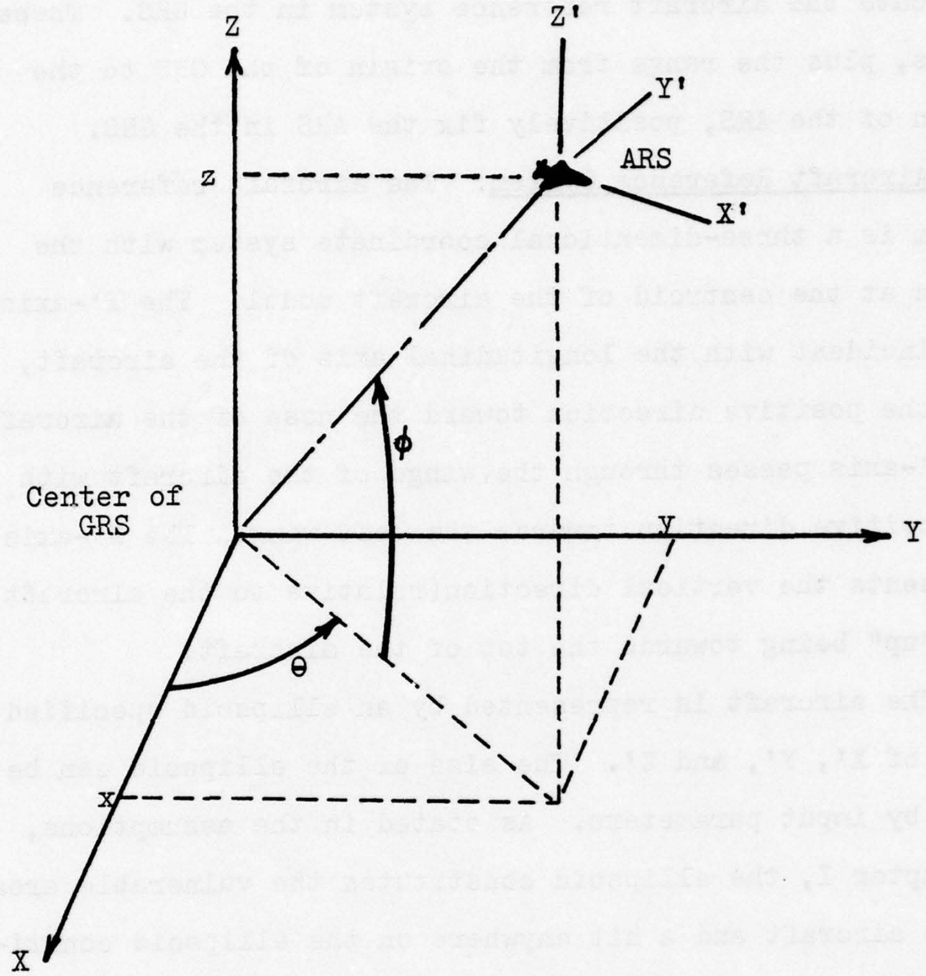
Coordinate Systems

Two coordinate systems are required to mathematically model the engagement. The primary coordinate system is the Ground Reference System(GRS). The gun is assumed to be at the origin of the GRS. The secondary coordinate system is the Aircraft Reference System(ARS), this system is used to define the shape of the aircraft(see figure 3-1).

Ground Reference System. The ground reference system is defined by three axes; X, Y, and Z. The X-Y plane represents the horizontal plane, with the positive Z axis representing the "up" vertical direction.

The elevation of the aircraft from the gun is measured from the X-Y plane towards the positive Z direction, and can take on values of 0° to 90°. The capability exist to have negative elevation angles, but is not used in this study.

Azimuth is measured from the positive X axis, counterclockwise towards the positive Y axis and can take on values



where: θ is the azimuth angle
ϕ is the elevation angle

Figure 3-1. Depiction of Coordinate Systems

from 0^{o} to 360^{o}. The azimuth and elevation angles are used to locate the aircraft reference system in the GRS. These angles, plus the range from the origin of the GRS to the origin of the ARS, positively fix the ARS in the GRS.

Aircraft Reference System. The aircraft reference system is a three-dimensional coordinate system with the origin at the centroid of the aircraft model. The X'-axis is coincident with the longitudinal axis of the aircraft, with the positive direction toward the nose of the aircraft. The Y'-axis passes through the wings of the aircraft with the positive direction towards the left wing. The Z'-axis represents the vertical direction(relative to the aircraft), with "up" being towards the top of the aircraft.

The aircraft is represented by an ellipsoid specified in terms of X', Y', and Z'. The size of the ellipsoid can be fixed by input parameters. As stated in the assumptions, of Chapter I, the ellipsoid constitutes the vulnerable area of the aircraft and a hit anywhere on the ellipsoid constitutes a kill. This definition of "kill" will be used throughout the remainder of this report.

The position of the aircraft is an input to the model as a function of time. The entire flight path of the aircraft must be input as straight(no turns) segments. At the start of each segment the following must be specified: time, position of the origin of the ARS in the X, Y, Z coordinates of the GRS, velocity(actually speed in this case) in each of the three directions, and roll angle. From this data,

Subroutine ACPOS interpolates to find the position of the aircraft at any time required during the simulation.

The roll, pitch, and yaw of the aircraft are only reflected in the GRS since the ARS is fixed to the aircraft. Positive roll is defined as clockwise rotation of the ARS about the X'-axis(right wing down). Yaw is rotation of the ARS about the Z'-axis, with positive being a left yaw. Pitch is defined as rotation of the ARS about the Y'-axis, with positive being "nose up". The roll parameter is input for each flight path segment, while yaw and pitch are calculated by ACPOS using the velocity components of the aircraft.

<u>Coordinate Transformation</u>. The use of two coordinate systems necessitates that a transformation be used so that coordinates in one system can be expressed as coordinates in the other system. This transformation was accomplished with the use Euler Angles. Since these angles do not provide a unique solution, the order of rotation used in this study is: yaw, pitch, and roll. The transformation is given by:

$$[T] = \begin{bmatrix} C_aC_b & -S_aC_g-S_gC_aS_b & S_aS_g-C_gC_aS_b \\ S_aC_b & C_aC_g-S_gS_aS_b & -S_gC_a-S_aS_bC_g \\ S_b & C_bS_g & C_bC_g \end{bmatrix} \qquad (3\text{-}1)$$

where: C and S represent Cosine and Sine respectively

Subscript a represents the yaw angle - alpha

Subscript b represents the pitch angle - beta

Subscript g represents the roll angle - gamma

Subroutine TRANS computes the elements of the transformation matrix and stores them for later use. The transformation is used by the program to transpose the shell coordinates in the GRS to coordinates in the ARS. The next section will describe the propagation of the shell in the GRS.

Shell Trajectory Calculations

The shell trajectory calculated by AAASIM uses the theory of ballistic trajectories to determine the shell position as a function of time. In the calculation of ballistic trajectories, drag is normally the only aerodynamic force considered. This is a valid assumption as long as the angle of attack of the shell is small. Ballistic projectiles may have an angle of attack when they exit the muzzle; however, for spin stabilized ballistic bodies the angle of attack will be near zero over the major portion of the trajectory. Thus, forces normal to the axis of symmetry can be neglected and the motion of the shell lies in a vertical plane defined by the initial velocity vector and the gravity vector.

Point mass equations of motion are used by AAASIM to calculate the position of the shell as a function of time, these equations are:

$$\ddot{X} = -\frac{RVAC_D}{2M}\dot{X} \tag{3-2}$$

$$\ddot{Y} = -\frac{RVAC_D}{2M}\dot{Y} \tag{3-3}$$

$$\ddot{Z} = -\frac{RVAC_D}{2M}\dot{Z} - G \tag{3-4}$$

where: $\ddot{X}, \ddot{Y}, \ddot{Z}$ are the magnitudes of the accelerations in the X, Y, Z directions respectively.

$\dot{X}, \dot{Y}, \dot{Z}$ are the magnitudes of the velocities in the X, Y, Z directions respectively.

V = the magnitude of the shell velocity.

R = the density of the air.

A = the cross-sectional area of the projectile perpendicular to the axis of symmetry.

C_D = the drag coefficient.

M = the mass of the projectile.

G = the magnitude of the acceleration due to gravity.

Calculation of Air Density. This study was made with the assumption that the effective range of the antiaircraft artillery shell would be no greater than 3000 meters. Using this assumption, and the additional assumption that the gun was at sea level, the air density (R) was approximated by the following linear regression (Ref 6:53):

$$R = 1.220449545 - .0001054811(Z) \qquad (3\text{-}6)$$

where: R = the density of the air at altitude Z

Calculation of Drag Coefficient. The coefficient of drag is an experimentially determined parameter that varies with the mach number of the shell. The mach number of the

shell was calculated using the speed of sound determined in the same way as the air density. The linear regressien used for the speed of sound is (Ref6 :53):

$$S = 340.4616 - .003872727(Z) \qquad (3\text{-}7)$$

where: S = the speed of sound at altitude Z

Once the mach number of the shell is determined, the appropiate C_D can be extracted Figure 3-2 which was taken from <u>Exterior Ballistic Data For Foreign 23mm and 57 mm Antiaircraft Systems - HITVAL I</u> (Ref 1:26). Projectile parameters of the 23mm shell were used exclusively in this study. AAASIM accounts for the different values of C_D in the subsonic, transonic, and supersonic regions by calculating the mach number of the shell and computing C_D as follows:

$$C_D = .154 \qquad M < .86$$

$$C_D = .300 \qquad .86 \leq M \leq 1.15$$

$$C_D = .52689344 - .06590164(M) \qquad M > 1.15$$

where: C_D = the coefficient of drag at mach number M

This method of determining C_D induces some discontinuity; however, it is expected that in the majority of cases the speed of the projectile will be supersonic causing the effect of this discontinuity to be minimal.

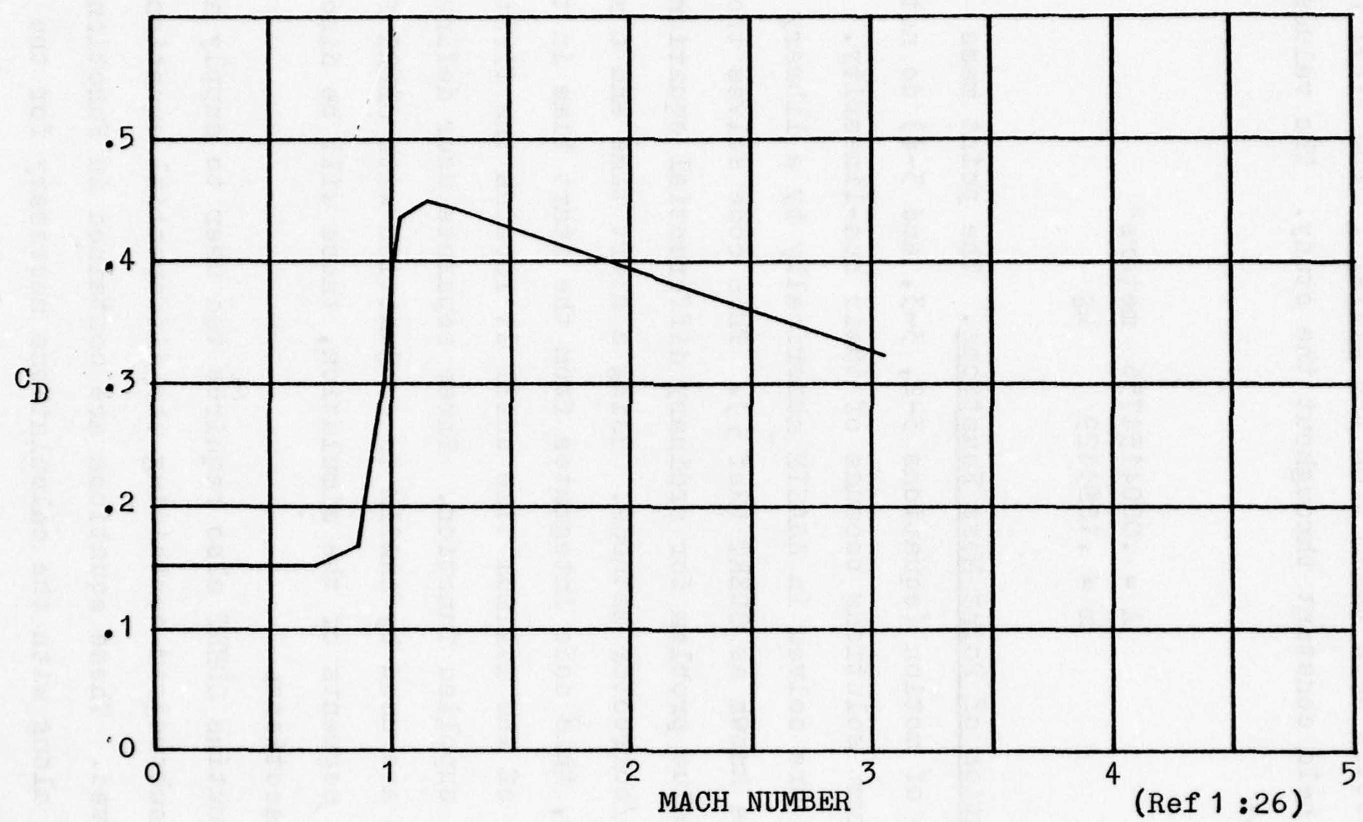


FIGURE 3-2. DRAG FORCE COEFFICIENT VS MACH NUMBER

23mm

The values for the cross-sectional area and the mass of the projectile were taken from the HITVAL I report(Ref 1:11) and were held constant throughout the study. The values used were:

$$A = .0004154756 \text{ meters}^2$$
$$m = .1885129 \text{ kg}$$

Solution of Point Mass Equations. The point mass equations of motion (equations 3-2, 3-3, and 3-4) do not have closed form solutions because of their non-linearity. The equations are solved in AAASIM numerically by a library computer code known as ODERT (Ref 5). This code solves the initial value problem for ordinary differential equations by predictor/corrector methods. Using a start time and a maximum time, this code integrates from the start time in the direction of the maximum time until it locates the first root of a user supplied function. Three separate user defined functions are used by AAASIM in conjunction with ODERT for different segments of the simulation, these will be discussed in later sections.

Subroutine ODERT also requires the user to supply a function subprogram containing the differential equations to be solved. These equations are contained in Function F of AAASIM along with the calculations necessary for the aerodynamic drag.

The first step in the simulation is to determine the mean azimuth and elevation aiming angles necessary to provide an intercept between the projectile and aircraft trajectories under error free conditions.

<u>Mean Theoretical Intercept Point</u>

The mean theoretical intercept point(MTIP) is defined as the intersection of the projectile and aircraft trajectories when tracking, aiming, and firing are done in an error free environment. The result of MTIP calculations is the determination of the mean azimuth and elevation angles for the gun and the mean time of flight of the shell. Once these values are determined, they are used in subsequent probability of hit calculations and shell distribution determination.

Subroutine MTIP is the routine in AAASIM that calculates the MTIP. This routine uses ODERT and the user supplied Function DR to calculate the trajectory of the shell. Function DR is the first derivative of the function specifying the distance between the shell and the centroid of the aircraft. ODERT integrates outward until it finds the first root of this function (the point where the distance reaches a minimum), it then returns the time of flight of the shell and the coordinates of the MTIP. Subroutine MTIP uses an iterative procedure, adjusting the aiming angles, until the minimum distance between the shell and the centroid of the aircraft is less than .05 meters. When this test is passed,

the aiming angles and the time of flight of the shell, to the MTIP, are returned to the calling routine. The calling routine can now superimpose the error sources of the system on these mean aiming angles and conduct a Monte Carlo sampling to calculate either the probabiltiy of hit or the shell distribution.

Monte Carlo Sampling Procedures

AAASIM uses Monte Carlo techniques to incorporate the identified error sources into the simulation. All of the error sources are assumed to be bivariate normally distributed with zero means. This study was also made assuming that the mean aiming angles were those calculated in Subroutine MTIP. Normal random deviates used in the error sampling were derived in Subroutine GGNOR, which was developed using the Box-Mueller transformation to transform uniform random deviates into normal deviates. The uniform random deviates were obtained from subroutine GGUB, a library routine in the International Mathematical and Statistical Library (IMSL). Subroutine GGNRM, also from the IMSL library, was used to sample from the aiming angle covariance matrix (Ref 2).

Aiming Angle Distribution. Variance in the aiming angles associated with the gun barrel are input to the model in the form of a covariance matrix. Monte Carlo samples are taken using this covariance matrix to determine random error sources for the aiming angles. These errors are then

Table 3-1. Ballistic Error Values

ERROR SOURCE	STD. DEVIATION
Muzzle Velocity Uncertainity	10% of the mean
Atmospheric Disturbance	.003 mils
Gun Jitter	.005 mils
Flight Roughness	$(.10\overline{V}_a T_f)$
Ballistic Dispersion	.0031 mils

(Ref 4 :28)

superimposed on the mean aiming angles. The next step is to consider the ballistic error sources.

Ballistic Error Sources. There are several other sources of error that affect the trajectory of the shell, these are the ballistic error sources. These errors are assumed to be bivariate normal with zero means and zero covariance. The ballistic error sources considered in this model are:

1. Muzzle velocity uncertainties,
2. Atmospheric disturbance,
3. Gun jitter,
4. Flight Roughness,
5. Ballistic dispersion.

These error sources and their nominal values were taken from P001, and are listed in Table 3-1. The impact of the error sources, and their values, will be discussed in Chapter IV.

Flight roughness takes into account the random movement of the aircraft from the time the shell is fired until the shell reaches the aircraft. This ballistic error source is calculated by randomly shifting the position of the aircraft, from the position determined by subroutine ACPOS. The amount of the shift is determined by random sampling a normal distribution with zero mean and a standard deviation of .010 (see Table 3-1) and multiplying the value obtained by the aircraft speed and the time of flight of the shell. The aircraft position is shifted in three independent directions based upon its speed in each of the three coordinate directions. The random position thus obtained is used in determining the probability of hit.

Each of the ballistic error sources is sampled using Monte Carlo techniques and superimposed upon the aiming angles previously determined from the aiming angle distribution. Once the azimuth and elevation angles are determined, for a particular replication, a shell distribution or a probability of hit is calculated for the imput sample size.

Probabiltiy of Hit Calculations

The probability of hit is calculated in Subroutine SAMPLE which uses Subroutine SHOT to determine whether a particular sample replication hits the aircraft, or not.

The azimuth and elevation angles of the shell, with respect to the GRS, are determined in Subroutine SAMPLE for each replication of a particular sample. These angles are

then used by Subroutine SHOT to propagate a shell toward the aircraft model. Subroutine ODERT is used again to calculate the trajectory of the shell. ODERT integrates from the time of fire until it finds a root of the user defined function, Function HIT.

<u>Function HIT</u>. The general equation for an ellipsoid is:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1 \qquad (3\text{-}8)$$

where: a, b, c are the semi-axis dimensions

In Function HIT the right hand side of this equation is changed to a variable and the derivative, with respect to time is calculated. For each time interval of the trajectory of the shell calculated by ODERT, the coordinates of the shell in the GRS are transformed to coordinates in the ARS using the transformation of equation 3-1 and substituted into the derivative of equation 3-8. As the shell comes closer to the ellipsoid, the value of the derivative will decrease and as the shell goes further from the ellipsoid the derivative will increase, changing sign in the process. At the point where the derivative changes sign, if the value of the left hand side of eq. 3-8 equals one or less, a hit is scored.

<u>Sample Probability of Hit</u>. A tally of whether each

sample is a hit or a miss is maintained by Subroutine SAMPLE. When all of the replications have been run, the sample probability of hit is simply the total number of hits divided by the sample size.

After the probability of hit is calculated, an IMSL routine BELBIN is used to calculate confidence bands for the estimated probability of hit. BELBIN computes a two-sided confidence interval for the probability of hit on any trial using the binominal distribution.

The primary output from this option of the simulation is an estimate of the probabiltiy of hit along with the confidence interval for the estimate. Another option of AAASIM is the determination of the shell impact distribution on the encounter plane.

Distribution of Shell Impact Points

Simulation model AAASIM has the capability to determine the shell distribution on the encounter plane for a particular sample. The first step in this process is to calculate the orientation of the encounter plane.

Encounter Plane Orientation. The encounter plane used in this study is a plane perpendicular to the relative velocity vector of the shell, with respect to the aircraft. The plane orientation was mathematically simulated by using the aircraft reference system, disregarding the ellipdoidal aircraft model.

Realizing that a transformation would have to be made

to transform the coordinates of the shell location into the encounter plane, the transformation developed earlier (see equation 3-1) could be used if pseudo yaw and roll angles could be determined. These pseudo yaw and roll angles are the azimuth and elevation mean aiming angles respectively. Using the mean azimuth and elevation aiming angles, the ARS could be rotated so that the X'-Z' plane was perpendicular to the relative velocity vector; with the Y'-axis pointing in the direction of the relative velocity vector. Once the orientation of the encounter plane is determined, it is held constant throughout a particular sample.

Impact Distribution. Subroutine SAMPLE uses the same procedures as were used for probabiltiy of hit calculations to superimpose the identified error sources upon the mean azimuth and elevation aiming angles. Subroutine DIST then propagates a particular shell along its trajectory, using ODERT, until the shell impacts the encounter plane. This position is then recorded and the next replication is simulated.

ODERT uses the Function YP to calculate the time of encounter plane impact. Function YP transposes the coordinates of the shell into ARS coordinates and calculates the Y' coordinate of the shell. When the Y' coordinate changes sign the impact point on the X'-Z' plane can be determined.

It is recognized that shell with different muzzle velocities and/or other trajectory errors will have different

times of flight. AAASIM accounts for this variation by moving the encounter plane along the aircraft velocity vector during each sample engagement.

Distribution Testing. Once a sample impact distribution has been determined, this distribution is tested to see it it is bivariate normal. The first step in this test is to calculate the vertical and horizontal means, variances, and the covariance. The covariance matrix, thus calculated, is then rotated using a Cholevsky decomposition to obtain two samples that can be independently tested for normality. Each of these samples is then tested for normality using a Chi-square goodness of fit test.

Simulation Output. The simulation outputs from the distribution of shell impact points option are the distribution parameters and the goodness of fit statistics. Distribution parameters include: the vertical and horizontal means, the vertical and horizontal variances, the covariance, and the correlation coefficient. The goodness of fit statistics include: the number of cells used in the test (an input option), the vertical Chi-square statistic, the horizontal Chi-square statistic, and the vertical and horizontal significant levels.

The next chapter will discuss the use of AAASIM, analyze the assumptions used in the modelling process and discuss the results of the planned simulation runs.

IV Analysis and Results

The purpose of this study, as stated earlier, was to determine the sensitivity of the probability of hit estimates to the parameters of the aiming angle distribution. The concept of development for AAASIM was to develop a model that provided a realistic simulation of the actual external and terminal ballistics of an antiaircraft artillery and aircraft engagement. The resultant model provides a more sophisticated simulation than model P001, primarily in the ballistic trajectory and target model portions. Monte Carlo techniques were used to superimpose the error sources, (affecting the external ballistics) on the aiming angle distribution so that either a shell distribution or a probability of hit could be determined. The use of Monte Carlo techniques induced several inherent problems into the analysis.

Problem Areas

Simulations using Monte Carlo techniques require that a value be assigned to each variable of the model and that the model be run once for each sample. Since each model run requires a finite amount of time on a computer, effective use of the computer becomes a major concern. The two major problems that have to be dealt with are the number of variables, and the sample size to be used.

Model Variables. There are 22 different variables considered in AAASIM. If n samples are required for a one vari-

able model then n^k samples are required if k parameters are involved. Since each of the variables, considered in AAASIM, is continuous in nature the number of model runs can quickly become overwhelming. To complicate the matter further, the number of samples for each model run must be considered.

Sample Size Considerations. The accuracy of point estimates for probabilistic variables is highly dependent upon sample size. If the point estimate desired is the probability of hit, the sample size can be estimated by approximating the binomial distribution with a normal distribution. This approximation is acceptable if the sample size is large and the actual probability of hit is not near zero or one. The equation used (Ref 3:131) is:

$$n = \frac{Z^2_{a/2}}{4d^2} \qquad (4\text{-}1)$$

where: n - is the required sample size

$Z_{a/2}$ - is the two-tailed standardized normal statistic for the probability desired

a - is the significance level desired

d - is the largest acceptable difference between the estimate and the true probability

Table 4-1 contains the approximate number of samples required for different levels of significance and different d's.

Table 4-1. Minimum Sample Size For Bernoulli Estimates

a	$Z_{a/2}$	d	n
.10	1.65	.1000	68
.10	1.65	.0100	6806
.10	1.65	.0010	680625
.10	1.65	.0001	68062500
.05	1.96	.1000	96
.05	1.96	.0100	9604
.05	1.96	.0010	960400
.05	1.96	.0001	96040000
.01	2.58	.1000	166
.01	2.58	.0100	16641
.01	2.58	.0010	1664100
.01	2.58	.0001	166410000

The sample sizes shown in Table 4-1 are valid when the actual probability of hit is not near zero. According to simulation runs, with zero mean aiming errors, this situation occurs when the range of the intercept is less than 1000 meters and the vulnerable area is reasonable value, i.e. in the order of 5-7 meters^2. For most engagements, however, the range will be in the order of 1000 - 2000 meters; in this range the probabilities of hit are typically on the order of .001 - .01. In this range of probabilities the sample sizes in Table 4-1 are somewhat conservative. A more realistic sample size for three significant figures is approximately 10000. Table 4-2 provides some typical ranges for accuracy

Table 4-2. Confidence Interval Bounds For 95% Confidence Level

n	P_k	Upper Limit	Lower Limit
1000	.001	.0056	.0000
1000	.010	.0183	.0048
1000	.100	.1202	.0821
5000	.001	.0023	.0003
5000	.010	.0132	.0074
5000	.100	.1087	.0918
10000	.001	.0018	.0005
10000	.010	.0121	.0081
10000	.100	.1060	.0942

of predictions that were calculated from the binominal distribution using an IMSL library routine, BELBIN.

The conclusion to be drawn from these two tables is that for low single shot probabilities of hit, extreme accuracy is only possible with very large sample sizes. Lack of accuracy can be detrimental when trying to distinguish between probabilities of hit that are close in value. It also means that to use computed values to several significant figures can be misleading.

The large sample sizes that are required for reasonable accuracy coupled with the vast number of possible variable values necessitated that restrictions be placed on the allowable range of the variables, and the sample sizes considered. Without these restrictions the computation time would have been unacceptable.

Typical AAASIM Run Times. The model designed for this study, AAASIM, was not necessarily designed for efficiency. However, when it was expedient to do so, time saving steps were taken. The simulation run time varied with the range to the intercept point, the option run, and the sample size. Typical run times for AAASIM were on the order of 40 - 70 seconds. When the run option desired is the computed probability of hit, AAASIM uses a method of selective sampling to lower the run time.

Selective Sampling. When the probability of hit is desired, each shell is propagated from firing until the shell either intercepts or misses the target. Once the azimuth and elevation angles of propagation are determined (by superimposing each of the error sources upon the input aiming angles) it can be analytically determined if the shell has any possibility of intercepting the target.

Since this model uses a shaped target, if the target position is known with certainty the firing angle necessary to just miss the target can be calculated. To account for different target positions, due to different shells times of flight, the semi-axes of the ellipsoidal shaped target are enlarged by three meters. The critical firing angle is then calculated analytically and used for a cut-off point for propagating shells. If the sample aiming angles are outside of these computed critical values the shell is not propagated, a miss is scored and the next sample is run.

This step bypasses the most time consuming portion of the simulation, thus saving the computation time in cases where the probability of a hit is zero.

In contrast, the simulation option that computes the shell distribution on the encounter plane must propagate each shell through to intercept, regardless of nearness of the impact point to the target. This option will always require the longest time to run.

The method of selective sampling used provides one computation time saving device; however, the major time saving device employed in the study was to reduce the variables considered and therefore reducing the replications.

Variable Value Selection

In order to make the study manageable, and to concentrate on the sensitivity analysis desired, a number of variables will be discussed in this section along with the possible ramifications of holding them constant.

Target Model. The target aircraft was modelled as a shaped target throughout the study. This is in contrast to modelling the target as a damage function, i.e. P001's diffused target model. The selection of a shaped target makes it possible to model the probability of hit as a series of Bernoulli trials, and one shot is either a hit or miss. One advantage of this method of target representation should be obvious, it reduces the computation of the probability of hit to simply scoring hits and misses. A second advantage

is the target can be moving throughout the simulated engagement, thus automatically accounting for differences in shell velocities and actual aircraft movement.

The disadvantage of this type of target model is that it is difficult to accurately represent the geometric shape of an aircraft. The shape used in AAASIM is an ellipsoid, and by selectively defining the dimensions of the semi-axes, the shape fairly accurately represents the body of an aircraft. The use of this model carries with it the implicit assumption that the wings, elevator, and vertical stablizer are not part of the aircraft vulnerable area. The vulnerable area of such a model can be controlled by varying the semi-axes dimensions. This was done in later comparisons of AAASIM results with those of P001.

A constant presented area, even though not necessarily accurate, is a desirable property when measuring the sensitivity of probability of hit to aiming angle distributions. A constant presented area is obtained by fixing the flight path of the aircraft and time to fire.

<u>Aircraft Flight Path Selection</u>. The aircraft flight path was kept straight and level for all replications of the simulation. The aircraft track was set parallel to the X-axis of the ground reference system, and the aircraft had a velocity only in the X direction. The Y-axis displacement was adjusted along with the Z - axis displacement to provide the desired range and elevation for a particular replication. Using a common start point, the location of the aircraft at

the time of intercept could be controlled by the time of fire.

Time of Fire. As mentioned earlier, a constant aircraft presented area is a desirable property when looking at the effect of varying the aiming angle distribution. This was accomplished in AAASIM by adjusting the time of fire so that the mean shell intercepted the target aircraft perpendicular to the longitudinal axis of the aircraft. In the model geometry, this point is defined as an azimuth aiming angle of 90°. When the intercept point is so located, the presented area of the target is not a function of the elevation angle; since with the ellipsoid model any elevation will have the same presented area as long as the cross-sectional area perpendicular to the longitudinal axis is circular. This is also the point of closest approach of the aircraft to the gun, and the point where the azimuth tracking rate is the greatest.

Modelling the engagement at the point where the azimuth tracking rate is the greatest might, at first, seen counter-productive;, however, this study does not model the tracking explicitly but assumes a given aiming angle distribution. Therefore, the errors that this method might induce in the tracking performance are not relevant to this study and are ignored.

A slight error is induced into the computations because the actual shell velocities vary about the mean shell velocity, and all shells will not see exactly the same presented area. This error is assumed to be very small and ignored.

Elevation Angles. There are an infinite number of possible elevation angles that could be considered. Since it would be impossible to consider all possible values, some restriction had to be placed on the elevation angle that would be used in the simulation. In the previous sections the location of the target aircraft had been so positioned that the presented area would not be a function of the firing elevation angle. The remaining consideration for the selection of possible elevation angles was the interrelation between elevation angle and the gravity drop of the projectile.

One of the objectives of this study was to determine the effect of gravity drop on the shell distribution. The possibility existed that gravity drop would distort the shell distribution and thus alter the probability of hit. Intuitively, the effects of gravity would be zero on the direction of a shell that traveled in the vertical direction, and would be the greatest on a shell that traveled in the horizontal direction. Therefore, an elevation angle of zero was selected so that the simulation results would yield a worst case situation of the effects of gravity on the shell distribution.

The actual situation modelled was to position the aircraft so that the elevation angle of the line of sight from the gun to the aircraft was zero. To accomplish this, the actual aiming angle had to be elevated slightly to compensate for the gravity drop. This was strictly a modelling consideration and the error induced should be negligible.

Intercept Range. The next possibility considered to reduce the range of variables was to limit the intercept ranges that would be used in the simulations. Range has two primary effects on probability of hit estimates. The first is that as the range of the shell increases, its speed decreases due to drag forces. As the speed of the shell decreases, its ability to damage the target (considering that it does hit the target) also decreases. Below some minimum value the shell is no longer a threat to the target aircraft. The second effect of range is that trajectory perturbations tend to cause a dispersion in the shell position from the mean shell position. Thus as the range increases the probability of hitting the target decreases.

These two effects of range combine to provide a practical maximum limit of the ranges to be considered. As far as a minimum range limit, the practical considerations are the altitude of the aircraft and the fact that as the intercept range gets extremely small(less than 500 meters), the error effects are negligible, resulting in very high probabilities of hit. The range of 1500 meters was chosen to make the most of the comparisons because it provided a compromise that allowed trajectory perturbations to have an effect on the trajectory and still provide probabilities of hit estimates that gives an appreciable number of expected hits in the number of shots sampled. Appreciable refers to earlier remarks on confidence bounds.

Ballistic Error Values. The values for the ballistic error sources considered in this study are standard deviations of circular normal distributions. Each error source is assumed to be circular normally distributed with a zero mean and a zero covariance. The error sources are also assumed to be independent of each other. The standard deviations used were the values used in P001 and were held constant throughout the study. Their values are shown in Table 3-1.

The ballistic error sources, as well as their values, are a major source of uncertainty in this analysis. The effects of this uncertainty will be discussed in later sections of this chapter.

Aiming Angle Distribution Parameters. The aiming angle distribution is assumed to be bivariate normal and defined by a covariance matrix and mean values of zero. The covariance matrix contains the remaining variables to be considered in the simulation. It is composed of a variance for the azimuth aiming angle, a variance for the elevation aiming angle, and a covariance. The sensitivity that is the subject of this study is the sensitivity of P_k estimates to these variances and covariance.

AMRL has modified P001 to account for more sophisticated determination of tracking performance. Using AMRL's version of P001, typical values of the aiming angle variances were computed. These values were in the range of .000001 to .000200. Assuming that this range is typical of the var-

iances that could be expected, this was the range used in this study. In order to reduce the number of simulation replications required, the distribution was assumed to be circular normal (both variances equal for each replication). In reality the variances would very seldom be equal, and if it is assumed that most aircraft flight paths would be more horizontal than vertical, a slight error will be induced in the vertical component of the shell distribution calculated.

The effect of this vertical error should be to underestimate the calculated probability of hit. Since this study is concerned with relative values, rather than absolute values, the effect of the error on the results of the study should be minimal.

The covariance effect was not readily apparent. To test for its effect, a series of runs were made where all values were held constant except for the covariance. The results of this test are shown in Table 4-3. These results indicated that the covariance could be taken to be zero without effecting the calculated probability of hit. Although small changes in the probability of hit were computed, these changes were well within the sampling noise of the model and could be considered statistically insignificant.

One point worth noting is that the P_k estimates seem to vary more for negative covariances than for positive covariances, especially for the higher correlation coefficients. If it is felt that the covariance should have more impact on P_k estimates, then further study might be required.

Table 4-3. Effects of Aiming Angle Covariance on P_k Estimates

Correlation Coefficient	Covariance	P_k	Confidence Level
.3333	.00001	.050	.037 - .065
.0333	.000001	.049	.036 - .064
.0167	.0000005	.048	.035 - .063
0	0	.047	.034 - .062
-.0167	-.0000005	.050	.037 - .065
-.0333	-.000001	.051	.038 - .066
-.3333	-.00001	.040	.029 - .054

Sample Size. The choice of sample size was determined primarily by computation time considerations. One additional consideration was that the model be able to differentiate between estimates for the probability of hit for the target vulnerable areas that would be used. A sample size of 1000 was taken as the standard to be used since this was the minimum number that could detect probabilities in the range of .001. As mentioned earlier, sample size is the driving factor determining accuracy of predictions, and since absolute accuracy was not the purpose of this study, sample sizes of 1000 proved adequate.

When using Monte Carlo techniques, the absolute values obtained are highly influenced by the seed used to initiate the random number generator. Since only relative values were required for this analysis, a constant value was selected for the starting seed. This same seed was used to initiate each

replication. By using the same seed, effects attributable to the random number stream could be eliminated, or at least pathologies would be reflected in all cases. The starting seed used in this study was 350.

This section has described the variables that were used in AAASIM, and where appropriate, given the values of the variables and the reasoning behind their selections. The next section will present the results of the analysis.

Results of Sensitivity Analysis

The purpose of this analysis is to examine what happens to probability of hit estimates when the aiming angle variances are varied. It should be noted that what is being varied is the variances of the aiming angles and not their standard deviations. If this fact is not kept in mind confusion in interpreting the results could occur. For example, an order of magnitude change in the variance from .00001 to .0001 is only a three fold or so change in the standard deviations. The factors that are going to effect the probability of hit estimates in this analysis are the variances of the aiming angles, the vulnerable area (presented area in this case) of the target, the range to the intercept point, and the ballistic error sources. The first step in the analysis will be to look at the effect of variance changes on the probability of hit estimates when all ballistic error sources are assigned a value of zero.

<u>Sensitivity Without Ballistic Errors</u>. In this first series of simulation runs the ballistic error sources were assigned values of zero, the intercept range was set at 1500 meters, the target elevation was set to zero degrees, and the presented area of the target was 62.8 meters^2.

A comment is in order at this point on the references to the presented area of the target. The target aircraft is modelled as an ellipsoid, where the dimensions of the semi-axes are input values to the model. The simulated anti-aircraft shell actually sees a volume rather than an area. The semi-axes dimensions input for this series of runs were: 10 meters for the X' semi-axis, and 2 meters for both the Y' and Z' axes. Viewed along a line which is perpendicular to the X' axis, the two-dimensional area presented is 62.8 m^2. This presented area is used in the analysis discussion as a convenience, although the actual volume was used in the simulation runs.

The curve shown in Figure 4-1, for zero ballistic errors, shows a very steep slope in the lower regions of variances, this slope decreases as the value of the variance increases. The extremes are obvious, when the variances are zero the probability of hit (in the zero mean error case) will be 1.0, and as the variance increases the probability of hit will tend toward zero. The important point to notice is the value of the slope in the region of the P001 computed aiming angle variance (for these conditions), .000045. Looking at this condition, a slight increase in the variance will

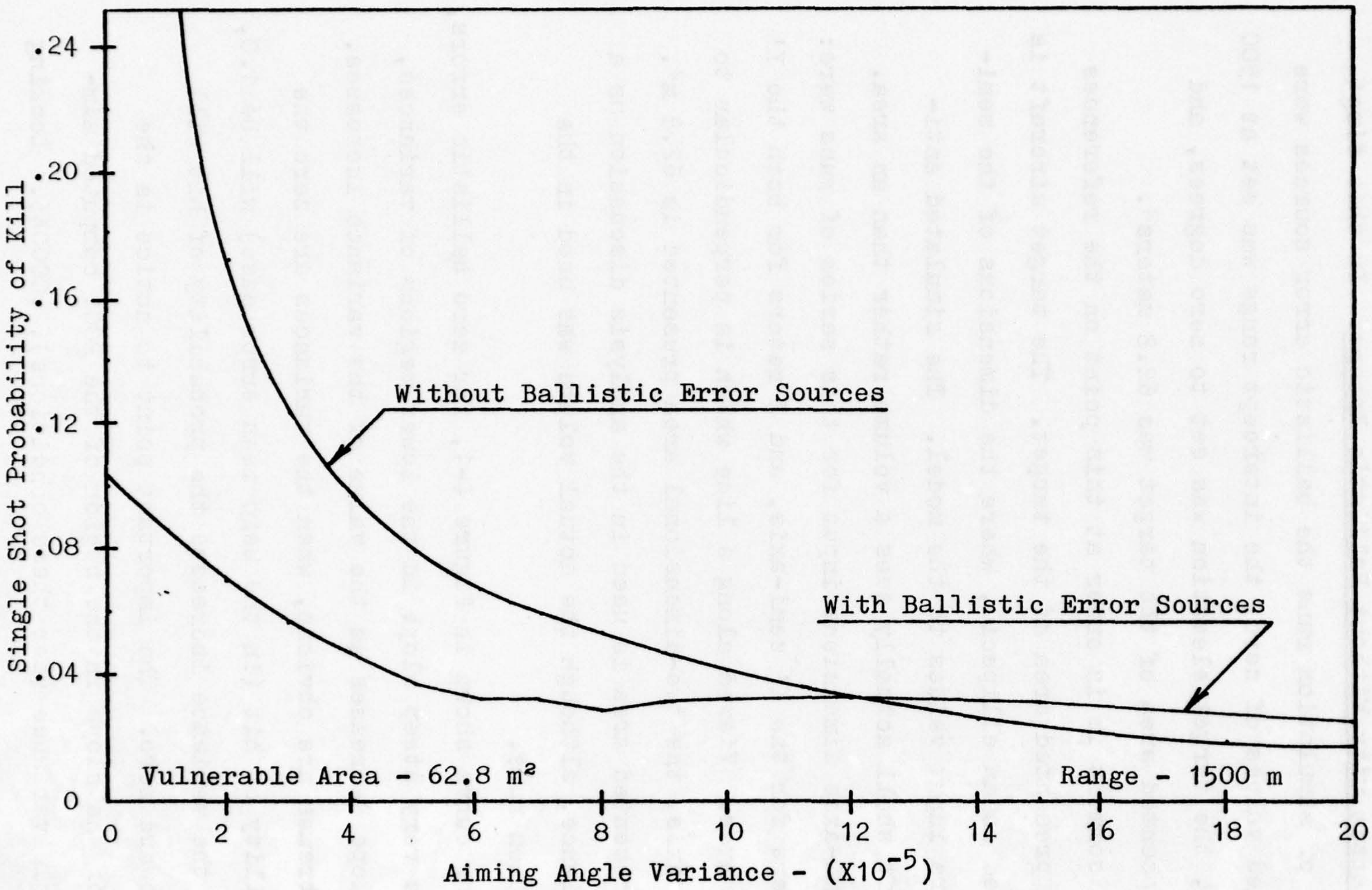


Figure 4-1. Comparison of Sensitivity Response Curves With and Without Ballistic Error Sources

produce a small decrease in the probability of kill (P_k) while an equal size decrease in the variance will produce a larger change in the P_k.

The key point in applying this analysis is the location of the current aiming angle variance. Its value will determine where on the response curve the user is working, and therefore the magnitude of the P_k change for a given change in the variance value. The next logical step is to add values for the ballistic error sources.

Sensitivity With Ballistic Errors. The response curve on Figure 4-1, labeled "With Ballistic Error Sources", was computed under the same conditions as the first response curve except that nominal values were assigned to all of the ballistic error sources. The values assigned are listed in Table 3-1. The result of the ballistic error sources addition was to decrease the overall P_k estimates, and more important, the response curve is much flatter.

The obvious result is that the addition of the ballistic error sources reduces the sensitivity of the P_k estimates to changes in the aiming angle variances. An effect not so obvious is that the resulting decrease in sensitivity is also a function of the magnitude of the P_k estimates.

Magnitude of P_k Estimates. The ballistic error sources and the errors in the aiming angles tend to cause a dispersion in the fired shells. Therefore, as the intercept range increases (all other model values held constant) the resulting P_k estimates will decrease. This result is shown

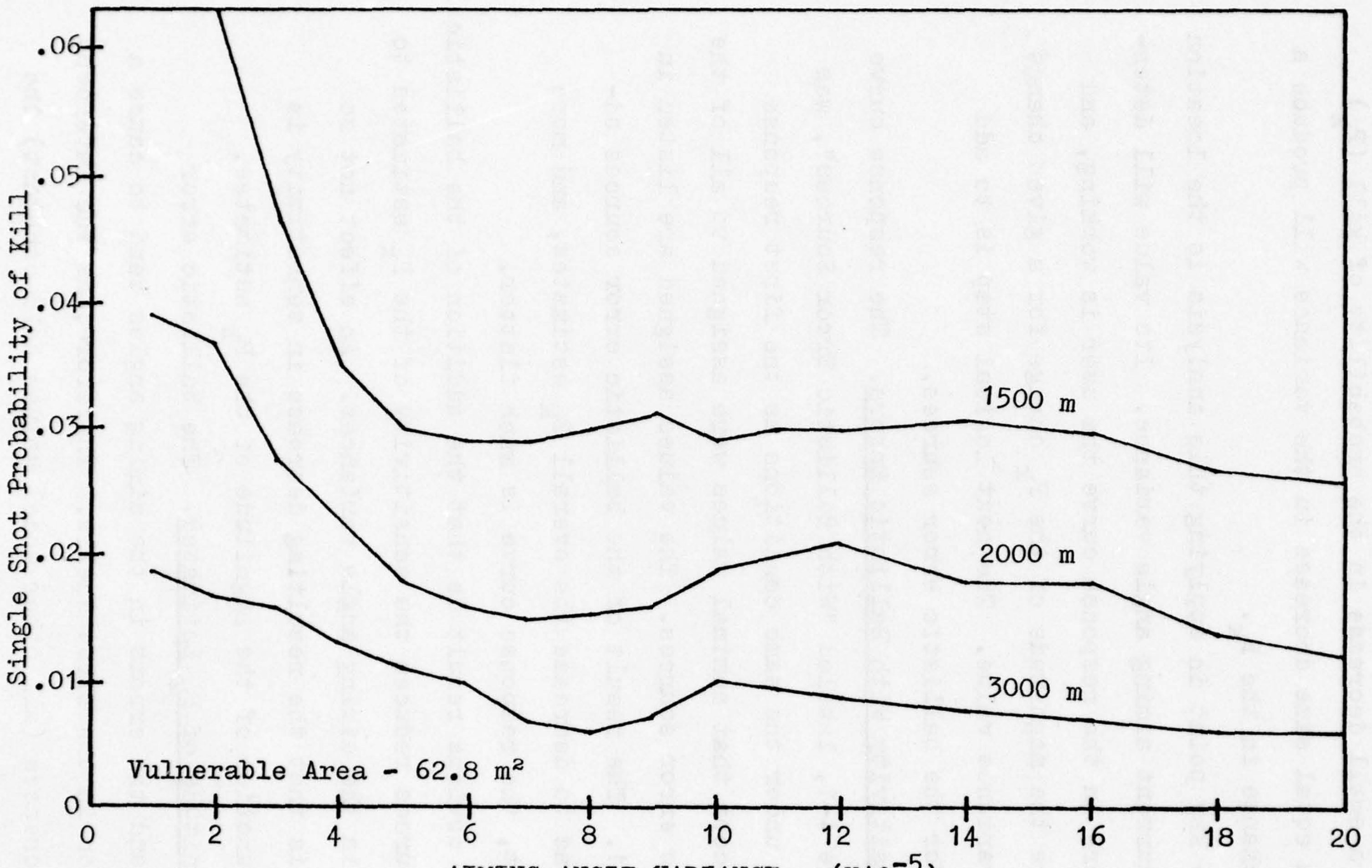


Figure 4-2. Affect of P_k Magnitude on P_k Rate of Change With Aiming Angle Variance

in Figure 4-2, note the change in scale from the previous figure. The only difference in the series of runs depicted in Figure 4-2 is the increase in the intercept range. The shell dispersion caused by the increase range resulted in a decrease in the P_k estimates computed. It is clear, from this series of runs, that as the magnitude of the P_k estimates decreases the response curves tend to flatten.

The conclusion to be drawn from this part of the analysis is that the sensitivity of the P_k estimates is a function of the magnitude of the P_k estimates themselves.

Effect of Ballistic Error Values. The ballistic error sources and the aiming angle variances both contribute to the dispersion of the fired projectile. All of these sources of dispersion are combined by Monte Carlo techniques. Two factors that will determine the sensitivity of the computed P_k estimates to any one of the error sources are: the sum of the ballistic error variances and the magnitude of the aiming angle variances. As the sum of the ballistic error sources variance increases, the overall effect of a change in the aiming angle variance will decrease. Likewise, if the sum of the ballistic error variances is significantly larger than the aiming variances, the effect of a change in the value of the aiming angle variances will have less of an impact on the computed P_k estimates.

These two factors point to a critical point in this study, the ballistic error sources and their values. The values, as well as the sources, were those currently used

Table 4-4. Changes in P_k Estimates for a Decrease In Variance From .00004 To .00002

Range	P_k for Variance .00004	P_k for Variance .00002	P_k Increase
500	.301	.351	+.050
1500	.035	.064	+.029
2000	.018	.037	+.019
3000	.013	.017	+.004

in P001. If the number of ballistic sources or their assumed values are invalid, the results of this analysis may change. The sources, themselves, are fairly well established, the values may be questioned but are not significantly suspect.

Significance of Results. The significance of the results of this analysis is that the sensitivity of P_k estimates is relative to the circumstances of the engagement. Whether P_k estimates are sensitive, or not, to changes in the aiming angle variances will depend on the particular response curve in question (defined by the conditions of the engagement) and the location on the response curve where the current aiming angle variance is positioned.

Various changes in P_k estimates that could be expected for a decrease in the aiming angle variances from .00004 to .00002 are listed in Table 4-4. A decrease in aiming angle variance of this magnitude equates to an angular standard deviation change of about 2 milliradians. The values in

Table 4-4 were extracted from Figure 4-2.

Referring back to Figure 4-2, the region of the response curve with the largest slope occurs below a variance of about .00005. Computed aiming angle variances from AMRL's modified version of P001 indicates that the current value of the variance is about .000045. This means that AMRL is presently working near the critical point of the response curve. Decreases in the variance would produce much more of a change in P_k estimates than an equal increase in the variance. The same type of analysis can be made for different ranges (or different vulnerable areas since both result in a decrease in the P_k estimate values), as the range increases the slope of the response curve decreases. Thus equal changes in the aiming angle variance will produce decreasing changes in the P_k estimates as the range of intercept increases.

<u>Confidence Intervals</u>. Model AAASIM also computes a confidence interval for each P_k estimate computed. Figure 4-3 illustrates a print out from AAASIM. At the bottom of the figure the confidence interval is shown for the computed P_k. The level of significance for the confidence interval is an input to the model and can be changed prior to each run, if so desired.

This confidence interval reflects the sampling noise level of the model and should be used to compare P_k estimates from simular runs. This noise level is a direct function of the sample size and the order of magnitude of the P_k estimate.

```
OPTION 5 - PROBABILITY OF HIT ON ELLIPSOIDAL SHAPED AIRCRAFT
                   GRAVITY DROP CONSIDERED

INPUT PARAMETERS
     TIME OF FIRE                                29.3900000 SECS,
     SAMPLE SIZE                                       1000
     AZIMUTH AIMING ANGLE STD.DEV.                 .0031623 RADIANS
     ELEVATION AIMING ANGLE STD.DEV.               .0031623 RADIANS
     AZIMUTH AIMING ANGLE VARIANCE                 .0000100
     ELEVATION AIMING ANGLE VARIANCE               .0000100
     AIMING ANGLE COVARIANCE                      0.0000000
     MUZZLE VELOCITY                            930.0000000    M/SEC
     MUZZLE VELOCITY STD.DEV.                     9.3000000    M/SEC
     BALLISTIC DISPERSION STD.DEV.                 .0031000 RADIANS
     ATMOSPHERIC DISTURBANCE STD.DEV.              .0030000 RADIANS
     GUN JITTER STD.DEV.                           .0050000 RADIANS
     FLIGHT ROUGHNESS COEFFICIENT                  .0100000

MEAN THEORETICAL IMPACT POINT (MTIP)
     TIME OF FLIGHT OF SHELL                       .5034443   SECS.
     TIME OF MEAN INTERCEPT                      29.9934443   SECS.
     POSITION OF AIRCRAFT AT MTIP
          X-COORDINATE                            -.3111311
          Y-COORDINATE                          500.0000000
          Z-COORDINATE                            0.0000000
     POSITION OF SHELL AT MTIP
          X-COORDINATE                            -.3159903
          Y-COORDINATE                          499.9948853
          Z-COORDINATE                            -.0001239
     RANGE OF SHELL                             499.9943857   METERS
     SHELL/AIRCRAFT DIST ERROR AT MTIP              .007779   METERS
     MTIP AZIMUTH ANGLE                           1.5714303 RADIANS
     MTIP ELEVATION ANGLE                          .0033491 RADIANS
     RELATIVE SPEED AT MTIP                     753.7555552    M/SEC

SIZE OF ELLIPSOIDAL AIRCRAFT
     X SEMI-AXIS                                 10.0000000   METERS
     Y SEMI-AXIS                                  2.0000000   METERS
     Z SEMI-AXIS                                  2.0000000   METERS
AIRCRAFT ATTITUDE RELATIVE TO GUN
     HEADING                                      0.0000000 RADIANS
     PITCH                                        0.0000000 RADIANS
     ROLL                                         0.0000000 RADIANS
NUMBER OF HITS                                          386
PROBABILITY OF HIT                                 .3860000
CONF. INTERVAL OF PROBABILITY OF HIT
     LOWER BOUND                                   .3557000
     UPPER BOUND                                   .4159733
CONFIDENCE LEVEL                                   .9500000
```

Figure 4-3. Sample AAASIM Printout Showing P_k Confidence Interval

To decrease the noise level of the model requires an increase in the sample size. Sample sizes for appropriate degrees of accuracy of predictions are contained Tables 4-1 and 4-2.

The next section will compare the results from AAASIM and P001. The region of comparison will be the response curves generated from each model, for like conditions.

P001 Response Curves

A modified version of P001 is presently used by AMRL to generate P_k estimates using a more sophisticated tracking model than the original P001 version. In this section a comparison is made between the response curves of P001 and the response curves of AAASIM. Prior to making this comparison, some observations will be made concerning assumptions used in P001 and the modelling methods that differ from AAASIM. One of the basic assumptions used in P001 is that the shell distribution on the encounter plane is bivariate normal. This assumption was tested in this study.

Bivariate Normal Assumption. Model P001 uses analytical methods to compute the P_k estimates for a particular run. To make these computations the assumption is made that the shell distribution on the encounter plane is bivariate normal. If the encounter plane is perpendicular to the mean shell velocity and the aiming distribution is bivariate normal, the shell distribution on the encounter plane can still be non-normal due to non-linear propagation of the shell. This situation is complicated further since the

encounter plane is actually perpendicular to the relative velocity vector of the shell and aircraft at the time of impact.

Model AAASIM was used to test this assumption by generating a shell distribution on the encounter plane and testing this distribution for normality. The method of testing is described in Chapter III and will not be described again here. The results of the test indicated that the distribution passed a Chi-square goodness of fit test when all of the ballistic error sources had their nominal values. A test run where the ballistic errors were assigned values of zero did not pass the goodness of fit test. The result of this test is shown in Figure 4-4. Several tests were run at various ranges, elevations, and azimuth angles and all of the distributions passed the tests except for the test shown in Figure 4-4.

The conclusion to be drawn is the normality assumption is valid as long as all of the ballistic error sources are assigned values near their nominal values. Erroneous results could occur if P001 runs were made considering only the errors in the aiming angles.

Gravity Drop Effects. Gravity drop was not considered in P001. One of the objectives of this study was to test the assumption that the effects of gravity drop on P_k estimates could be neglected. Gravity drop actually causes a flatter distribution than one without gravity drop. However, this effect is small at normal ranges of intercept,

```
              DISTRIBUTION OF SHELL IMPACT POINTS
OPTION 3 INTERCEPT PLANE PERPENDICULAR TO RELATIVE VELOCITY
                  GRAVITY DROP CONSIDERED

INPUT PARAMETERS
    TIME OF FIRE                            29.3900000 SECS,
    SAMPLE SIZE                                   1000
    AZIMUTH AIMING ANGLE STD.DEV.             .0054772 RADIANS
    ELEVATION AIMING ANGLE STD.DEV.           .0054772 RADIANS
    AZIMUTH AIMING ANGLE VARIANCE             .0000300
    ELEVATION AIMING ANGLE VARIANCE           .0000300
    AIMING ANGLE COVARIANCE                  0.0000000
    MUZZLE VELOCITY                        930.0000000   M/SEC
    MUZZLE VELOCITY STD.DEV.                 0.0000000   M/SEC
    BALLISTIC DISPERSION STD.DEV.            0.0000000 RADIANS
    ATMOSPHERIC DISTURBANCE STD.DEV.         0.0000000 RADIANS
    GUN JITTER STD.DEV.                      0.0000000 RADIANS
    FLIGHT ROUGHNESS COEFFICIENT             0.0000000

MEAN THEORETICAL IMPACT POINT (MTIP)
    TIME OF FLIGHT OF SHELL                   .5100018   SECS.
    TIME OF MEAN INTERCEPT                  30.0000018   SECS.
    POSITION OF AIRCRAFT AT MTIP
        X-COORDINATE                          .0003556
        Y-COORDINATE                       354.0000000
        Z-COORDINATE                       354.0000000
    POSITION OF SHELL AT MTIP
        X-COORDINATE                          .0003756
        Y-COORDINATE                       353.8305571
        Z-COORDINATE                       354.1109249
    RANGE OF SHELL                         500.5319735  METERS
    SHELL/AIRCRAFT DIST ERROR AT MTIP          .155524  METERS
    MTIP AZIMUTH ANGLE                       1.5707953 RADIANS
    MTIP ELEVATION ANGLE                      .7850947 RADIANS
    RELATIVE SPEED AT MTIP                 752.5297348   M/SEC

DISTRIBUTION PARAMETERS
    VERTICAL MEAN                              .202184  METERS
    VERTICAL STD.DEV.                        2.7375905  METERS
    HORIZONAL MEAN                           -.1197836  METERS
    HORIZONTAL STD.DEV.                      1.9731400  METERS
    VERT/HORZ POSITION COVARIANCE           -1.2503675
    CORRELATION COEFFICIENT                  -.2325242

GOODNESS OF FIT TO NORMAL DISTRIBUTION
    NUMBER OF CATAGORY CELLS                        20
    VERTICAL CHI-SQUARE STATISTIC          160.3500000
    VERTICAL SIGNIFICANCE LEVEL               .0000000
    HORIZONTAL CHI-SQUARE STATISTIC        127.2400000
    HORIZONTAL SIGNIFICANCE LEVEL             .0000000
```

Figure 4-4. Normality Test of Shell Distribution
Without Ballistic Error Sources

i.e. less than 2000 meters. Simulation runs of AAASIM that generated a shell distribution showed a negligible effect of gravity drop on the distribution. Figures 4-5 and 4-6 show identical runs except that Figure 4-5 does not consider gravity and Figure 4-6 does consider gravity. The differences between both the means and the variances of these two runs is in the second decimal place.

Gravity effects were also tested to determine the effect on P_k computations. The method of testing was to change the elevation of the target aircraft. As discussed in Chapter III, the effects of gravity will be greatest (on direction) with an elevation angle of zero and smallest with an elevation angle of 90^o. This test consisted of two sets of runs: one at an elevation angle of zero degrees, and one at 45^o elevation. The result of this test is shown in Figure 4-7. The different response curves are well within the confidence interval of each other. Thus a definite conclusion can not be made, except to note the trend. The direction of the trend is intuitive since the zero elevation run had overall lower P_k estimates and a steeper slope. Zero elevation should have the greatest effect on the shell distribution, thus causing the lower P_k estimates. The effects of gravity appear to be negligible in the intercept ranges of interest in this study.

The final observation to be made concerning P001 is the use of the empirical relationship to represent the shell trajectory, versus the point mass differential equation

```
              DISTRIBUTION OF SHELL IMPACT POINTS
OPTION 3 INTERCEPT PLANE PERPENDICULAR TO RELATIVE VELOCITY


INPUT PARAMETERS
     TIME OF FIRE                                  27.5500000 SECS,
     SAMPLE SIZE                                         1000
     AZIMUTH AIMING ANGLE STD.DEV.                   .0054772 RADIANS
     ELEVATION AIMING ANGLE STD.DEV.                 .0054772 RADIANS
     AZIMUTH AIMING ANGLE VARIANCE                   .0000300
     ELEVATION AIMING ANGLE VARIANCE                 .0000300
     AIMING ANGLE COVARIANCE                        0.0000000
     MUZZLE VELOCITY                              930.0000000   M/SEC
     MUZZLE VELOCITY STD.DEV.                       9.3000000   M/SEC
     BALLISTIC DISPERSION STD.DEV.                   .0031000 RADIANS
     ATMOSPHERIC DISTURBANCE STD.DEV.                .0030000 RADIANS
     GUN JITTER STD.DEV.                             .0050000 RADIANS
     FLIGHT ROUGHNESS COEFFICIENT                    .0100000

MEAN THEORETICAL IMPACT POINT (MTIP)
     TIME OF FLIGHT OF SHELL                        2.4539799   SECS.
     TIME OF MEAN INTERCEPT                        30.0039799   SECS.
     POSITION OF AIRCRAFT AT MTIP
          X-COORDINATE                               .7958838
          Y-COORDINATE                            1500.0000000
          Z-COORDINATE                              0.0000000
     POSITION OF SHELL AT MTIP
          X-COORDINATE                               .7971135
          Y-COORDINATE                            1500.0020920
          Z-COORDINATE                               .0223438
     RANGE OF SHELL                               1500.0023039   METERS
     SHELL/AIRCRAFT DIST ERROR AT MTIP                .022470   METERS
     MTIP AZIMUTH ANGLE                             1.5702649 RADIANS
     MTIP ELEVATION ANGLE                            .0000149 RADIANS
     RELATIVE SPEED AT MTIP                       459.5011547   M/SEC

DISTRIBUTION PARAMETERS
     VERTICAL MEAN                                   .2340029   METERS
     VERTICAL STD.DEV.                             13.0330463   METERS
     HORIZONAL MEAN                                 -.1340143   METERS
     HORIZONTAL STD.DEV.                           13.3453584   METERS
     VERT/HORZ POSITION COVARIANCE                -96.7063055
     CORRELATION COEFFICIENT                        -.5355792

GOODNESS OF FIT TO NORMAL DISTRIBUTION
     NUMBER OF CATAGORY CELLS                              20
     VERTICAL CHI-SQUARE STATISTIC                 14.3400000
     VERTICAL SIGNIFICANCE LEVEL                     .7327109
     HORIZONTAL CHI-SQUARE STATISTIC               20.3200000
     HORIZONTAL SIGNIFICANCE LEVEL                   .3755584
```

Figure 4-5. Shell Distribution Without Gravity Effects

```
               DISTRIBUTION OF SHELL IMPACT POINTS
OPTION 3 INTERCEPT PLANE PERPENDICULAR TO RELATIVE VELOCITY
                     GRAVITY DROP CONSIDERED

INPUT PARAMETERS
   TIME OF FIRE                                27.5500000 SECS,
   SAMPLE SIZE                                       1000
   AZIMUTH AIMING ANGLE STD.DEV.                 .0054772 RADIANS
   ELEVATION AIMING ANGLE STD.DEV.               .0054772 RADIANS
   AZIMUTH AIMING ANGLE VARIANCE                 .0000300
   ELEVATION AIMING ANGLE VARIANCE               .0000300
   AIMING ANGLE COVARIANCE                      0.0000000
   MUZZLE VELOCITY                            930.0000000   M/SEC
   MUZZLE VELOCITY STD.DEV.                     9.3000000   M/SEC
   BALLISTIC DISPERSION STD.DEV.                 .0031000 RADIANS
   ATMOSPHERIC DISTURBANCE STD.DEV.              .0030000 RADIANS
   GUN JITTER STD.DEV.                           .0050000 RADIANS
   FLIGHT ROUGHNESS COEFFICIENT                  .0100000

MEAN THEORETICAL IMPACT POINT (MTIP)
   TIME OF FLIGHT OF SHELL                      2.4433552   SECS.
   TIME OF MEAN INTERCEPT                      30.0033552   SECS.
   POSITION OF AIRCRAFT AT MTIP
      X-COORDINATE                               .7710307
      Y-COORDINATE                          1500.0000000
      Z-COORDINATE                              0.0000000
   POSITION OF SHELL AT MTIP
      X-COORDINATE                               .7753077
      Y-COORDINATE                          1500.0105537
      Z-COORDINATE                               .0233827
   RANGE OF SHELL                           1500.0103645   METERS
   SHELL/AIRCRAFT DIST ERROR AT MTIP             .025586   METERS
   MTIP AZIMUTH ANGLE                           1.5702791 RADIANS
   MTIP ELEVATION ANGLE                          .0152114 RADIANS
   RELATIVE SPEED AT MTIP                     459.7034298   M/SEC

DISTRIBUTION PARAMETERS
   VERTICAL MEAN                                  .234379  METERS
   VERTICAL STD.DEV.                           13.0371918  METERS
   HORIZONTAL MEAN                              -.1234378  METERS
   HORIZONTAL STD.DEV.                         13.8360492  METERS
   VERT/HORZ POSITION COVARIANCE              -96.9135902
   CORRELATION COEFFICIENT                      -.5329110

GOODNESS OF FIT TO NORMAL DISTRIBUTION
   NUMBER OF CATAGORY CELLS                            20
   VERTICAL CHI-SQUARE STATISTIC               14.2400000
   VERTICAL SIGNIFICANCE LEVEL                   .7395250
   HORIZONTAL CHI-SQUARE STATISTIC             20.1200000
   HORIZONTAL SIGNIFICANCE LEVEL                 .3373900
```

Figure 4-6. Shell Distribution With Gravity Effects

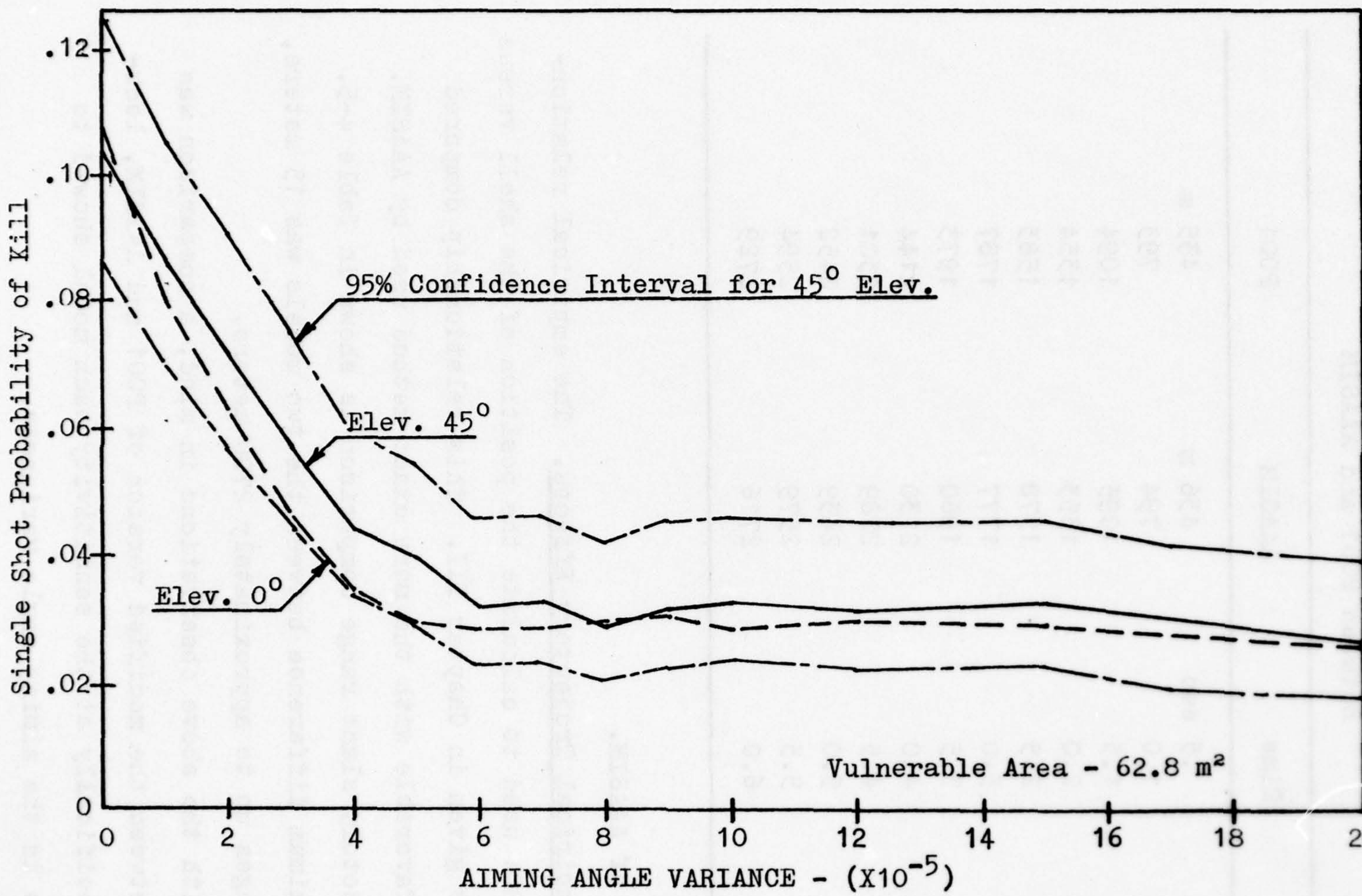


Figure 4-7. Probability of Kill vs. Aiming Angle Variance
Range 1500 meters
Gravity Drop Effects

Table 4-5. Comparison of Slant Range Calculations Between P001 and AAASIM

Time	AAASIM	P001
.5 sec	436 m	435 m
1.0	794	793
1.5	1095	1094
2.0	1353	1354
2.5	1578	1583
3.0	1777	1787
3.5	1960	1973
4.0	2130	2144
4.5	2289	2301
5.0	2439	2452
5.5	2579	2594
6.0	2716	2729

method of AAASIM.

Empirical Trajectory Effects. The empirical relationship P001 used to calculate the position of the shell versus time is given in Chapter III. This relationship compared quite favorable with the more exact method used by AAASIM. A trajectory slant range comparison is shown in Table 4-5. The maximum difference between the two models was 15 meters, for ranges up to approximately 2700 meters.

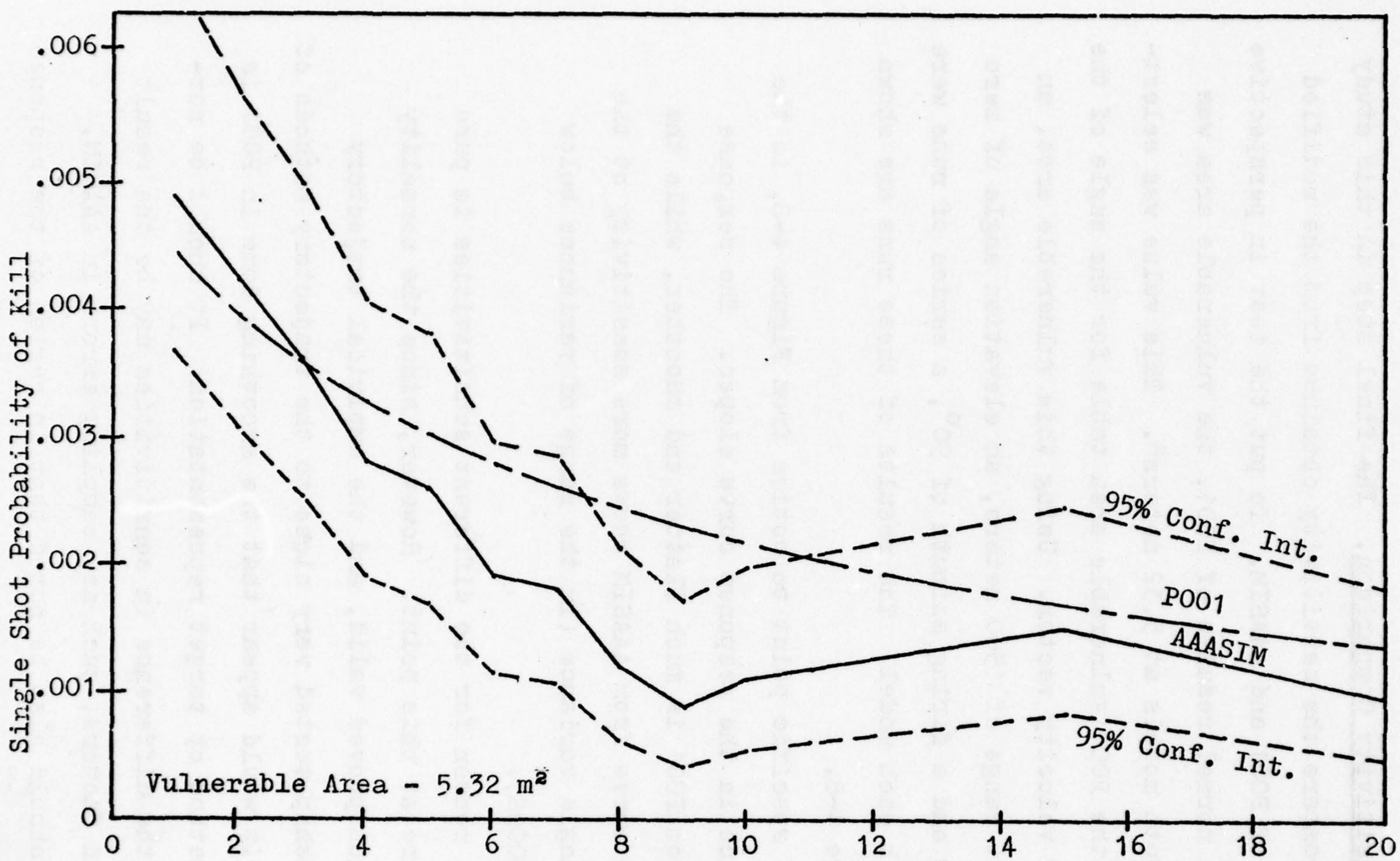
With the above observations in mind, a comparison was made between the modified version of P001 and AAASIM, looking specifically at the sensitivity each model showed to changes in the aiming angle variances.

Sensitivity Comparison. The final step in this study was to compare the sensitivity obtained from the modified version of P001 and AAASIM. To put the test in perspective with the normal results of P001, the vulnerable area was set in both models at 5.32 meters^2. This value was selected from the P001 vulnerable area table for the angle of the relative velocity vector. Using this vulnerable area, an intercept range of 1500 meters, an elevation angle of zero degrees, and a firing azimuth of 90°, a series of runs were made with each model. The results of these runs are shown in Figure 4-8.

The specific point to notice from Figure 4-8, is the difference in the response curve slopes. The response curve from P001 is much flatter and smoother, while the response curve from AAASIM shows more sensitivity ot the aiming angle variance (in the range of variances below about .00008).

The reason for the different sensitivities is pure conjecture at this point. However, since the normality assumption proved valid, and the empirical trajectory relationship tested very close to the trajectory methods of AAASIM, it would appear that the smoothing dome in P001 is in the method ot target representation. It should be noted that the difference in sensitivities may be the result of hidden factors, such as, sampling errors in AAASIM.

It should also be noted that in region of the response curves where the current aiming angle variance is located



AIMING ANGLE VARIANCE - (X10^{-5})

Figure 4-8. Probability of Kill vs. Aiming Angle Variance
Range 1500 meters
Comparison of P001 and AAASIM

(.000045 computed from P001) the P_k estimates are also close, within the 95% confidence interval computed for the data of AAASIM.

Observing the data in Figure 4-8, it would appear feasible that more sensitivity could be obtained from AAASIM than from P001 for otherwise identical cases. Whether the difference in sensitivity is significant,or not, would be up to the user. A summary of the above analysis along with conclusions and recommendations will be presented in Chapter V.

V Conclusions and Recommendations

This study has concentrated on analyzing the sensitivity of probability of kill estimates, derived from antiaircraft artillery and aircraft engagements, to parameters of a supplied aiming angle distribution. The supplied distribution was assumed to be bivariate normal with zero means. The study was sponsored by AMRL because preliminary analysis of the tracking performance of human trackers has shown that significant variations in tracking capabilities did not seem to produce equally significant variations in the probability of kill estimates.

In the process of AMRL's analysis, a more sophisticated tracking submodel than the original P001 version was developed. This improved submodel was incorporated into P001 and the remaining portions of P001 were used to calculate the probability of kill estimates. When the resulting estimates did not show the expected degree of response to tracking variations the possibility existed that lack of precision in the remaining submodels of P001 could be producing a smoothing effect. Three areas of concern were identified for further study: the fire control director submodel, the exterior ballistics submodel, and the terminal effects submodel. The exterior and terminal ballistics submodels are the point of this study. The fire control director submodel is the subject of another study.

In order to calculate probability of kill estimates,

given an aiming angle distribution, a dynamic model (AAASIM) was developed that used point mass equations of motion to calculate the trajectory of the shell, and an ellipsoidal shaped target was used to model the aircraft. Monte Carlo sampling techniques were used to sample the random errors from the aiming angle distribution and the ballistic error sources.

The analysis methods used in this study and the results of the analysis have been presented in previous chapters. This chapter will discuss the conclusions that can be drawn from the analysis and provide some recommendations for improving the response of P001.

Conclusions

The first point to be made concerning conclusions that can be drawn from this analysis is that, by design, the analysis was very narrow in scope. The purpose of the study was to concentrate specifically on the sensitivity of the probability of kill estimates to the variances of the aiming angle distribution. To accomplish this purpose, many variables in the model were fixed in value. The values used were believed to be reasonable and consistent with the real world system; however, if these constant values should change the literal output from AAASIM will change. By limiting the scope of the analysis the desired sensitivity could be pinpointed, but the literal output is of little value. The real value of the model is the relative results obtained

and the trends that developed.

<u>Probability of Kill Sensitivity</u>. The most interesting heuristic result of this analysis was the shape of the response curve obtained when P_k was plotted versus the aiming angle variance. The response curves consistently showed two distinctive slopes. Initially, when the aiming angle variances was low, the curves had a relatively steep slope. This initial slope remained fairly constant until the aiming variance increased to a range roughly equavalent to the sum of the variances of the ballistic error sources. At this point the response curves tended to flatten to a shallow slope. See Figure 4-8, which is included here again for convenience.

The interesting range of aiming angle variances occurred between 0.0 and .00020. After the latter value, the response curve sloped gradually downward, eventually resulting in P_k values very near zero. Relatively sensitivity will then be highly dependent upon the position of the nominal value of the variance. If the nominal aiming angle variance is on the steeper portion of the curve, the sensitivity of P_k estimates will considerably more than if the nominal value is on the flatter portion of the curve.

The factors that appeared to drive the P_k sensitivity were: the magnitude of the P_k values themselves, the vulnerable area of the target, and the sum of the ballistic error sources. The affect of these factors can be seen from a simple analytical model.

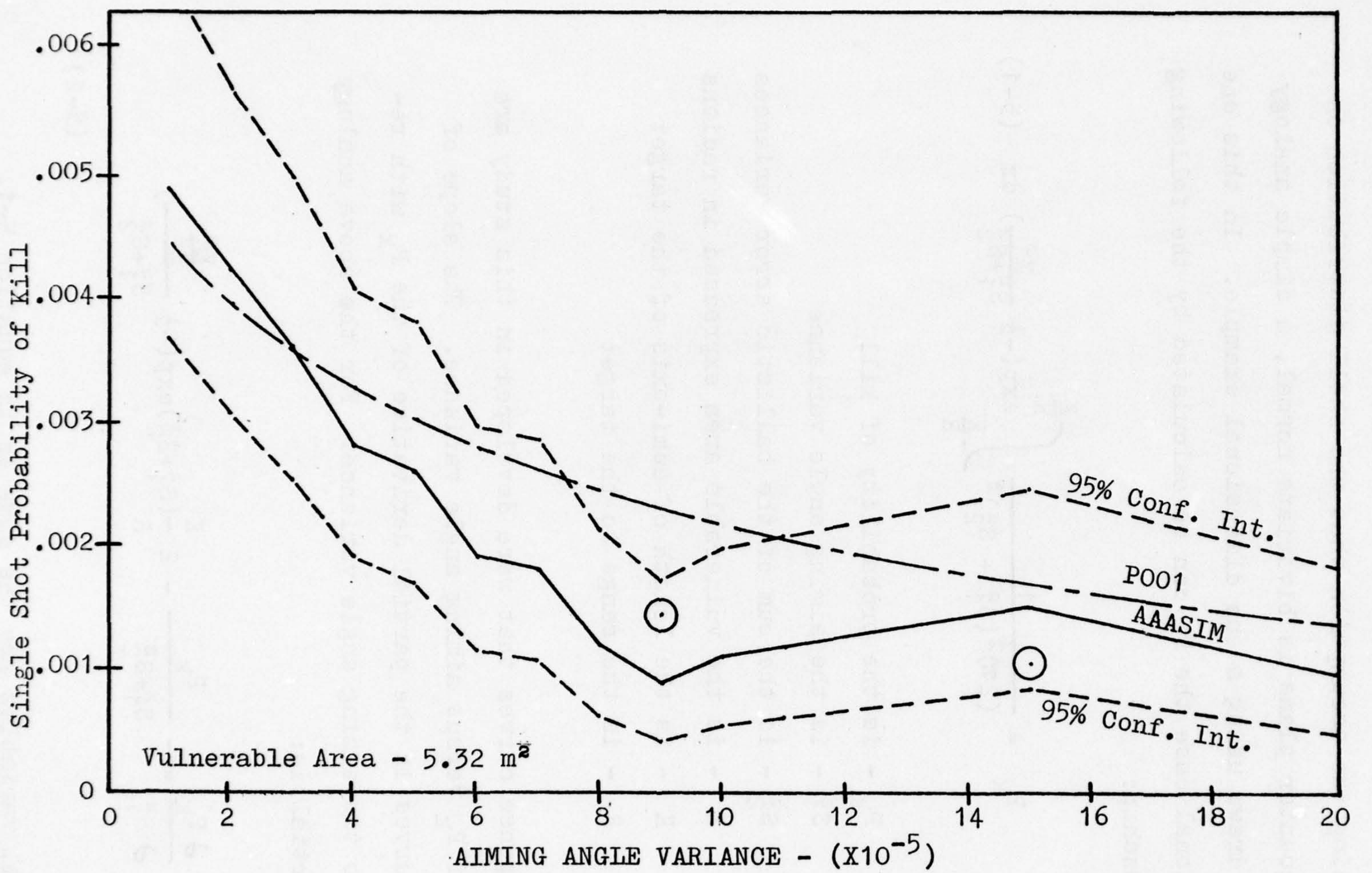


Figure 4-8. Probability of Kill vs. Aiming Angle Variance
Range 1500 meters
Comparison of P001 and AAASIM

Using the assumption that the shell distribution on the encounter plane is bivariate normal, a simple analogy can be drawn using a one dimensional example. In this one dimensional case the P_k can be calculated by the following relationship:

$$P_k = \frac{1}{(2\pi)^{\frac{1}{2}}(S_1^2 + S_2^2)^{\frac{1}{2}}} \int_{-\frac{K}{R}}^{\frac{K}{R}} \exp\left(-\frac{1}{2}\,\frac{X^2}{S_1^2+S_2^2}\right) dx \quad (5\text{-}1)$$

where: P_k - is the probability of kill

S_1^2 - is the aiming angle variance

S_2^2 - is the sum of the ballistic error variances

X - is the vulnerable area expressed in radians

K - is the length of semi-axis of the target

R - is the range to the target

The response curves that were developed in this study are plots of P_k versus aiming angle variance. The slope of these curves is the partial derivative of the P_k with respect to the aiming angle variance. For the above analogy this partial is:

$$\frac{\partial P_k}{\partial S_1^2} = -\frac{P_k}{S_1^2+S_2^2} - 2\,\frac{K}{R}(S_1^2+S_2^2)\exp\left(-\frac{1}{2}\,\frac{\frac{K^2}{R^2}}{S_1^2+S_2^2}\right) \quad (5\text{-}2)$$

where all variables are the same as in equation 5-1.

Analyzing the terms in equation 5-2, for the one dimensional case, provides some indications of the factors that will affect the slope of the response curves. The first term proves to be the most significant, it shows that the slope will become more negative as the magnitude of the P_k increases (for a constant denominator) and alternately, as the denominator increases in magnitude the slope will decrease. If the magnitude of the ballistic error variances remains constant, then the slope decreases as the aiming angle variance increases. This corresponds directly to the observed findings of this study.

The second term is a function of the vulnerable area ($\frac{K}{R}$) as well as the aiming angle variance and the variance of the ballistic error sources. As the vulnerable area increases, the slope will become more negative (have a steeper slope). An increase in the vulnerable area will also increase the P_k calculated from equation 5-1. This analysis shows the impact of the vulnerable area on the shape of the response curves.

The above analogy provides some analytical interpretation for the findings of this study with the exception of the apparent flattening of the response curves generated in this study. This apparent flattening could very well be caused by sampling error since it occurred in areas of low P_k and small perturbations in the number of hits in this region could cause apparent abnormal behavior in the response curves.

To make a more accurate determination of the exact shape of the response curves generated by AAASIM, several replications should be made for each point on the curves. The values from these replications could then be averaged and the mean of the points plotted. Since this was not possible in this study (because of time constraints) it can not be ascertained unequivocally that AAASIM produces more sensitivity than P001. However, it does appear from the trends of the response curves produced by AAASIM, that the methods employed by P001 could be dampening some of the sensitivity.

The more important result of this analysis has been the highlighting of driving factors in determining the sensitivity of P_k estimates to aiming angle variances, and the realization that on some portions of the response curves the sensitivity is very low.

Covariance Affects. The covariance of the aiming angle distribution was tested to ascertain what impact it had on the P_k estimates. The covariance was varied plus and minus three orders of magnitude, using a nominal value of the variance of .00003. The results of this test indicated that the P_k range for the plus covariances was between .047 and .050, and the minus covariances was .047 and .040. Although the actual values are not statistically significant, there appears to be a trend indicating more affect for negative covariances than for positive covarinaces.

The conclusion drawn in this analysis, is that, generally the covariance can be ignored in the P_k calculations; however, if the covariance is negative and on the same order of magnitude of the aiming angle variance further study of this effect might be required. The implicit assumption used, by taking the covariance term to be zero, is that the aiming angle errors are independent.

<u>Normality Test</u>. Model P001 calculates P_k estimates using the assumption that the shell distribution on the encounter plane is bivariate normal. One of the objectives of this study was to test this assumption. The assumption was tested and found to be valid. Over the range of aiming angle variances tested, the shell distribution passed a Chi-square goodness of fit test for all values of the variances. This test was made using all of the ballistic error sources set at their nominal values. One exception to the above results occurred when the shell distribution was determined without the ballistic error sources, this case did not the goodness of fit test.

The conclusion that can be drawn is that the ballistic error sources, in conjunction with the aiming angle errors, produce a central limit effect on the distribution resulting in a distribution which is normal. Using a model, such as P001, without the ballistic error sources would produce a shell distribution that was not normal and thus could invalidate the results.

Gravity Affects. The model developed for this study included the affect of gravity in the trajectory calculations. Two tests were conducted to ascertain the affect of gravity on the P_k estimates calculated. The first test used identical conditions, except for gravity, at an elevation angle of zero. The response curves generated from these two runs overlapped. The second test examined the case where one run was made at an elevation of 0^o and another run at 45^o, all other conditions the same. The difference in the two response curves was still insignificant; however, the run at 45^o showed a possible trend of increased P_k estimates.

It is conceivable that at extended ranges the affects of gravity could be significant, but for the ranges considered in this study the affects of gravity can be ignored.

Trajectory Calculations. A comparison was made between the slant range of the shell calculated by AAASIM, using the more exact trajectory model, and the slant range calculated by P001 using its empirical relationship. For slant ranges up to 2700 meters, the results of the P001 empirical relationship was found to be within 15 meters of results of AAASIM. Considering tha fact that this is a slant range and not a miss distance, the differences between the two methods is considered to be insignificant.

This conclusion, in addition to the conclusion on gravity affects, verifies that P001's empirical trajectory model is adequate for the intercept ranges considered valid for this study.

In light of the conclusions drawn above, some recommendations can now be made on possible modifications to P001 and areas requiring further study.

Recommendations

One of the big drawbacks to AAASIM was the computer time required to run a sample. The majority of the computation time was required by ODERT to solve the differential equations of the trajectory. The results of the analysis indicate that this time could be saved by using the empirical relationship of P001 (or a simular one) to calculate the trajectory of the shell.

Additionally, it appears that the difference between the response curves generated by P001 and AAASIM is caused by the method of sampling the error sources and/or the method of representing the target. A logical extention of this study would be to modify P001 to use Monte Carlo sampling techniques and a shaped target target.

The vulnerable area of the target could be varied by internally adjusting the dimensions of the shaped target using the vulnerable area values obtained from the already incorporated vulnerable area table. The Monte Carlo sampling would replace the present expected value method used by P001. Another alternative would be to use a systematic sampling method in place of the Monte Carlo method.

The resulting modification would obviously take longer to run than the present version of P001, but not as long as

AAASIM. The obvious tradeoff would be whether the more accurate response that could possibly be obtained would be worth the additional computation time to the user.

Overall the results obtained from P001 compare quite favorably with the results from AAASIM, and it can not be determined with any degree of certainty that the modelling methods of AAASIM are any better than P001. Obviously, the operational efficiency of P001 is far superior to that of AAASIM and any gain in sensitivity would be costly in computation time.

Bibliography

1. Campbell, Frederick M.,Jr. and Joseph W. Kochenderfer. Exterior Ballistic Data for Foreign 23MM and 57MM Antiaircraft Systems - HITVAL I. BRL-MR-2626. Aberdeen Proving Ground, MD: USA Ballistic Research Laboratories, May 1976. (AD B011645).

2. International Mathematical and Statistical Libraries. Computer Subroutine Codes. International Mathematical and Statistical Libraries, Inc, 6th Ed. , July 1977.

3. Mize, Joe H. and J. Grady Cox. Essentials of Simulation. Englewood Cliffs: Prentice-Hall, Inc., 1968.

4. Severson, James and Thomas McMurchie. Antiaircraft Artillery Simulation Computer Program - AFATL Program P001, Volume II. Analyst Manual. Technical Note 4564-16-73. Eglin AFB, FL:Air Force Armament Laboratory, September 1973.

5. Shampine, L.F. and M.K. Gordon. "Using Subroutine ODERT To Solve Ordinary Differential Equations," Sandia Mathematical Program Library. SAND - 77-1441. Sandia Laboratories, Albuquerque, NM., May 1978.

6. U.S. Standard Atmosphere, 1976. NOAA-S/T 76-1562. National Oceanic and Atmospheric Administration, Washington, D.C.,October 1976.

Supplementary Bibliography

1. Helgert, H. J. "On the Computation of Hit Probability," Opns. Res.19: 668-684, 1971.

2. HITVAL System Scoring Test Report, Volume I. AFSWC-TR-74-29. Kirtland AFB, NM: Air Force Special Weapons Center, May 1975. (AD B004561).

3. McNolty,F. "Kill Probability When The Weapon Bias Is Randomly Distributed," Opns. Res.10: 693-701, 1962.

4. Paris, W. B. and R.M. Baldwin. EVADE I, Evaluation of Air Defense Effectiveness, Computer Program Utilization Report. Technical Memorandum No. 40. Aberdeen Proving Ground, MD: USA Ballistic Research Laboratories, June 1969. (AD 855146).

5. Ross, J. A. An Analysis and Comparison of Three Aircraft Attrition Models. Paper P-967. Arlington, Va: Institute for Defense Analysis, July 1974. (AD B002784).

6. Severson, James and Thomas McMurchie. Antiaircraft Artillery Simulation Computer Program - AFATL Program P001, Volume I. User Manual. Technical Note 4565-16-73. Eglin AFB, Fl: Air Force Armanent Laboratory, September 1973.

7. Transue, John R. SIMFIND Models of Anti-Aircraft Gun Systems. Research Paper P-564. Arlington, Va: Institute For Defense Analyses, March 1970. (AD A024468).

8. Transue, John R. Feasibility of a Test of Probability of Hit By Antiaircraft Guns. WSEG Report 190. Arlington, Va: Institute For Defense Analyses, August 1972. (AD A017521).

Appendix A

Source Listing of AAASIM

```
      PROGRAM AAASIM(INPUT,OUTPUT,TAPE1,TAPE2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     AAASIM - ANTIAIRCRAFT ARTILLERY MONTE CARLO MODEL                C
C     BY MAJOR ERROL C. WILKINS - GST-79M                              C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION COVAR(3),COV(3),STATZ(3),STATX(3)
C
C     INITIALIZE ERROR VALUES FOR SUBROUTINE ODERT
C
      RELER=.000001
      ABSER=.000001
      REROOT=.000001
      AEROOT=.000001
C
C     READ INPUT DATA
C
      DO 10 I=1,21
      DO 10 J=1,8
      READ*,ACLFT(I,J)
      IF(ACLFT(I,1).GT.9999.)GO TO 20
10    CONTINUE
20    READ*,SEMIA,SEMIB,SEMIC
      READ*,VMUZEL,SMIN
30    READ*,TIME,AZIM,ELEV,NSAM,IOPT,ISEED,KATS
      IF(EOF(5LINPUT).NE.0)GO TO 70
      READ*,(COVAR(I),I=1,3)
      DO 35 I=1,3
35    COV(I)=COVAR(I)
      LSEED=ISEED
      READ*,(SIG(L),L=1,5)
C
```

```
C     INITIALIZE COUNTERS FOR DISTRIBUTION PARAMETERS
C
      SZSQ=0.0
      SXSQ=0.0
      SUMX=0.0
      SUMZ=0.0
      SXZ=0.0
      IPK=0
C
C     CALCULATE MEAN THEORETICAL INTERCEPT POINT
C
      CALL SECOND(A)
      CALL MTIP(TIME,VMUZEL,AZIM,ELEV,SMIN)
      CALL SECOND(B)
50    TOF=TIP(8)-TIME
      ELM=ELEV
      AZM=AZIM
C
C     RUN MODEL FOR EITHER PROBABILITY OF HIT OR DISTRIBUTION
C
      CALL SECOND(D)
      CALL SAMPLE(COVAR,ISEED,NSAM,AZIM,ELEV,VMUZEL,SMIN,TIME,IPK)
      CALL SECOND(E)
      IF((IOPT.GT.3).OR.(KATS.LT.3))GO TO 60
C
C     CALCULATE CHI-SQUARE GOODNESS OF FIT TO NORMAL DISTRIBUTION
C
      CALL NTEST(KATS,STATZ,STATX)
      REWIND 2
C
C     OUTPUT RESULTS OF SIMULATION
C
60    CALL PRINT(NSAM,TIME,IPK,VMUZEL,STATZ,STATX,KATS,COV)
      WRITE(1)TIME,TOF,TIP(8),NSAM,IOPT,(B-A),(E-D)
      GO TO 30
70    REWIND 1
```

```
      PRINT 200
80    ICHECK=1
90    READ(1)TIME,TOF,TIP(8),NSAM,IOPT,BA,ED
      IF(EOF(1).NE.0)GO TO 100
      PRINT 300,TIME,TOF,TIP(8),NSAM,IOPT,BA,ED
      PRINT 400,LSEED
      ICHECK=ICHECK+1
      IF(ICHECK.LT.6)GO TO 90
      PRINT 200
      GO TO 80
100   PRINT 200
200   FORMAT(1H1,////)
300   FORMAT(/,T14"TIME OF FIRE",T53,F13.7T69"SECS",/,T18"TIME OF FLIGHT
     1",T53,F13.7T69"SECS",/,T18,"TIME OF IMPACT",T53,F13.7T69"SECS",/,T
     218"NUMBER OF SAMPLES",T59,I7/,T18"OPTION",T65,I1/,T18"CENTRAL PROC
     3ESSER TIME",/,T22"FOR MTIP CALCULATIONS",T53,F13.7T69"SECS",/,T22"
     4FOR DISTRIBUTION OR PROB. OF HIT",T53,F13.7T69"SECS")
400   FORMAT(T18,"STARTING RANDOM SEED",T59,I7)
      STOP"  END OF RUN......"
      END
```

```
      SUBROUTINE SAMPLE(COVAR,ISEED,NSAM,AZIM,ELEV,VMUZEL,SMIN,TIME,IPK)
C
C     SUBROUTINE SAMPLE CONDUCTS THE MONTE CARLO SAMPLING FOR
C     EITHER PROBABILITY OF HIT CALCULATIONS OR THE
C     DETERMINATION OF THE SHELL DISTRIBUTION ON THE
C     ENCOUNTER PLANE
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION COVAR(3),WKVEC(2),RVEC(1000,2),R(6)
      CALL GGNRM(ISEED,NSAM,2,COVAR,1000,RVEC,WKVEC,IER)
      IF(IOPT.LE.3)WRITE(2)TIME,NSAM
      DO 30 J=1,NSAM
      AZ=AZIM+RVEC(J,1)
      EL=ELEV+RVEC(J,2)
      CALL GGNOR(ISEED,6,R)
      R4=R(4)
      DO 20 I=1,3
      AZ=AZ+SIG(I)*R(I)
   20 EL=EL+SIG(I)*R(I)
      VMUZ=VMUZEL+SIG(5)*R(5)
      IF(IOPT.LT.4)GO TO 40
      CALL SHOT(TIME,AZ,EL,VMUZ,SMIN,IHIT)
      IPK=IPK+IHIT
      GO TO 30
   40 CALL DIST(AZ,EL,TIME,VMUZ)
   30 CONTINUE
      IF(IOPT.GE.4)RETURN
      WRITE(2) SXSQ,SZSQ,SUMX,SUMZ,SXZ
      RETURN
      END
```

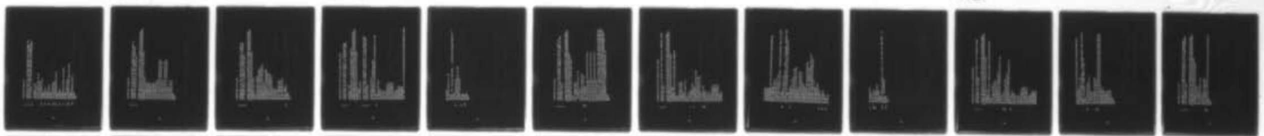
```
      SUBROUTINE ACPOS(T,ACL)
C
C     SUBROUTINE ACPOS CALCULATES BY TABLE INTERPOLATION THE
C     POSITION OF THE AIRCRAFT AS A FUNCTION OF TIME.  IT ALSO
C     CALCULATES THE ROLL,PITCH,AND YAW ANGLES OF THE AIRCRAFT
C     REFERENCED TO THE GUN COORDINATE SYSTEM.
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HTTT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R
      DIMENSION ACL(6),T(2),AC(2)
      DO 10 I=1,101
      IF((T(1).GE.ACLFT(I,1)).AND.(T(1).LT.ACLFT(I+1,1)))GO TO 20
10    CONTINUE
20    DO 30 J=1,3
      ACL(J)=ACLFT(I,J+1)+ACLFT(I,J+4)*(T(1)-ACLFT(I,1))
30    ACL(J+3)=ACLFT(I,J+4)
      AC(1)=ACL(4)
      AC(2)=ACL(5)
      ALPHA=AZMUTH(AC)
      IF((ACL(4).EQ.0.0).AND.(ACL(5).EQ.0.0))GO TO 40
      BETA=ATAN(ACL(6)/(SQRT(ACL(4)**2+ACL(5)**2)))
      GO TO 80
40    IF(ACL(6))50,60,70
50    BETA=-ATAN(1.0)*2.
      GO TO 80
60    BETA=0.0
      GO TO 80
70    BETA=ATAN(1.0)*2.
80    GAMMA=ACLFT(I,8)
      RETURN
      END
```

AD-A069 447 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH--ETC F/G 15/3
SENSITIVITY OF AIRCRAFT ATTRITION ESTIMATES TO AIMING DISTRIBUT--ETC(U)
MAR 79 E C WILKINS

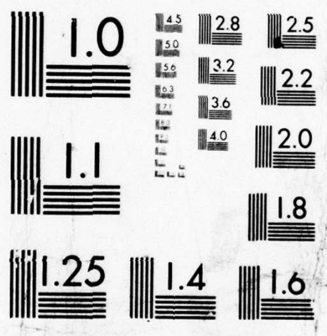
UNCLASSIFIED AFIT/GST/MA/79M-6 NL

2 OF 2

AD
A069447



END
DATE
FILMED
7--79
DDC



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

```
      FUNCTION AZMUTH(X)
C
C     FUNCTION AZMUTH IS A UTILITY FUNCTION THAT PETURNS THE
C     AZIMUTH OF THE AIRCRAFT FROM THE GUN IN A RANGE OF
C     ZERO DEGREES TO 360 DEGREES MEASURED COUNTERCLOCKWISE.
C
      DIMENSION X(6)
      PI=ATAN(1.0)*4.
      IF(X(1).NE.0.0)GO TO 40
5     IF(X(2))10,20,30
10    AZMUTH=(PI*3.0)/2.0
      RETURN
20    AZMUTH=0.0
      RETURN
30    AZMUTH=PI/2.0
      RETURN
40    IF(X(1))50,5,90
50    IF(X(2))60,70,80
60    AZMUTH=PI+ATAN(X(2)/X(1))
      RETURN
70    AZMUTH=PI
      RETURN
80    AZMUTH=PI/2.+ATAN(-X(1)/X(2))
      RETURN
90    IF(X(2))100,20,110
100   AZMUTH=PI*3.0/2.0-ATAN(X(1)/X(2))
      RETURN
110   AZMUTH=ATAN(X(2)/X(1))
      RETURN
      END
```

```
      SUBROUTINE TRANS(NT)
C
C     SUBROUTINE TRANS IS ROTATIONAL TRANSFORMATION USED TO
C     TRANSFORM COORDINATES IN THE AIRCRAFT COORDINATE
C     SYSTEM TO COORDINATES IN THE GUN SYSTEM.
C
      COMMON IGG,ACCEL(6),RELER,ABSEF,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION TS(3),TC(3)
      TS(1)=SIN(ALPHA)
      TC(1)=COS(ALPHA)
      TS(2)=SIN(BETA)
      TC(2)=COS(BETA)
      TS(3)=SIN(GAMMA)
      TC(3)=COS(GAMMA)
      C(1,1)=TC(1)*TC(2)
      C(1,2)=-TS(1)*TC(3)-TS(3)*TC(1)*TS(2)
      C(1,3)=TS(1)*TS(3)-TC(3)*TC(1)*TS(2)
      C(2,1)=TS(1)*TC(2)
      C(2,2)=TC(1)*TC(3)-TS(3)*TS(1)*TS(2)
      C(2,3)=-TS(3)*TC(1)-TS(1)*TS(2)*TC(3)
      C(3,1)=TS(2)
      C(3,2)=TC(2)*TS(3)
      C(3,3)=TC(2)*TC(3)
      RETURN
      END
```

```
      SUBROUTINE F(T,S,DS)
C
C     SUBROUTINE F IS THE FUNCTION ROUTINE THAT IS USED BY
C     SUBROUTINE ODERT TO SOLVE THE DIFFERENTIAL EQUATIONS
C     OF MOTION FOR THE TRAJECTORY OF THE SHELL
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION S(6),DS(6)
      G=0.0
      IF(IGG.NE.0)G=9.80665
      RHO=1.2204495-.00010548*S(3)
      SOUND=340.4615-.003872727*S(3)
      VEL=SQRT(S(4)**2+S(5)**2+S(6)**2)
      AMACH=VEL/SOUND
      CD=.52689344-.06590164*AMACH
      IF(AMACH.LT.1.15)CD=.300
      IF(AMACH.LT.0.86)CD=.154
      HO=.0011019819*RHO*CD
      DS(1)=S(4)
      DS(2)=S(5)
      DS(3)=S(6)
      DS(4)=-HO*VEL*S(4)
      DS(5)=-HO*VEL*S(5)
      DS(6)=-HO*VEL*S(6)-G
      DO 10 L=1,6
10    ACCEL(L)=DS(L)
      RETURN
      END
```

```
      SUBROUTINE SHOT(TIME,AZ,EL,VMUZEL,SMIN,IPK)
C
C     SUBROUTINE SHOT CALCULATES WHETHER A PARTICULAR SHOT
C     REPLICATION EITHER HITS OR MISSES THE AIRCRAFT MODEL.
C
      EXTERNAL F,HIT
      DIMENSION S(6),ACL(6),WORK(726),IWORK(5),DMY(6)
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      T=TIME+TOF
      CALL ACPOS(T,ACL)
C
C     GIVEN THE AIRCRAFT POSITION, DETERMINE IF THIS REPLICATION
C     WILL POSSIBLY PASS WITHIN 3 METERS OF THE MAJOR SEMI-AXIS
C     EXTREME POINT.
C
      IF(ISS.EQ.1)GO TO 10
      DHIT=ATAN((SEMIA+3.0)/RANGE(ACL,DMY))
      IF((ABS(AZ-AZM).GT.DHIT).OR.(ABS(EL-ELM).GT.DHIT))GO TO 100
10    IGG=0
      IF(IOPT.EQ.5)IGG=1
      VEL=VMUZEL
      S(1)=0.0
      S(2)=0.0
      S(3)=0.0
      VXY=VEL*COS(EL)
      S(4)=VXY*COS(AZ)
      S(5)=VXY*SIN(AZ)
      S(6)=VEL*SIN(EL)
      IFLAG=1
      TINV=TIME
      TOUT=TIME+10.
      RELERR=RELER
      ABSERR=ABSER
      CALL ODERT(F,6,S,TINV,TOUT,RELERR,ABSERR,IFLAG,WORK,IWORK,HIT,RERO
```

```
      1OT,AEROOT)
       IF(TINV.LE.TOUT)GO TO 30
       IPK=0
       PRINT ,"*****INVALID SOLUTION  -  INTEGRATION REACHED TOUT*****"
       RETURN
 30    CALL ACPOS(TINV,ACL)
       IF(CLV(S,ACL).LT.SMIN)GO TO 100
       IF(HITT-1.)20,20,100
 20    IPK=1
       RETURN
 100   IFK=0
       RETURN
       END
```

```
      FUNCTION HIT(T,S,DS)
C
C     FUNCTION HIT IS A USER DEFINED FUNCTION USED BY ODERT TO
C     DETERMINE THE POINT OF CLOSEST APPROACH OF THE SHELL TO THE
C     ELLIPSOID TARGET
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION S(6),ACL(6),DS(6),R(6), SFR(3)
      NT=1
      TF=T-(TIP(8)-TOF)
      CALL ACPOS(T,ACL)
        IF(SIG(4).EQ.0.0)GO TO 20
      DO 10 I=1,3
      SFR(I)=SIG(4)*TF*ACL(I+3)
10    ACL(I)=SFR(I)*R4+ACL(I)
20    CALL TRANS(NT)
      XP=C(1,1)*(S(1)-ACL(1))+C(2,1)*(S(2)-ACL(2))+
     1C(3,1)*(S(3)-ACL(3))
      YP=C(1,2)*(S(1)-ACL(1))+C(2,2)*(S(2)-ACL(2))+
     1C(3,2)*(S(3)-ACL(3))
      ZP=C(1,3)*(S(1)-ACL(1))+C(2,3)*(S(2)-ACL(2))+
     1C(3,3)*(S(3)-ACL(3))
      DXP=C(1,1)*(S(4)-ACL(4))+C(2,1)*(S(5)-ACL(5))+C(3,1)*(S(6)-ACL(6))
      DYP=C(1,2)*(S(4)-ACL(4))+C(2,2)*(S(5)-ACL(5))+C(3,2)*(S(6)-ACL(6))
      DZP=C(1,3)*(S(4)-ACL(4))+C(2,3)*(S(5)-ACL(5))+C(3,3)*(S(6)-ACL(6))
      HIT=(XP*DXP/(SEMIA**2))+(YP*DYP/(SEMIB**2))+(ZP*DZP/(SEMIC**2))
      HITT=(XP/SEMIA)**2+(YP/SEMIB)**2+(ZP/SEMIC)**2
      RETURN
      END
```

```
      SUBROUTINE MTIP(TIME,VMUZEL,AZ,EL,SMIN)
C
C     SUBROUTINE MTIP CALCULATES THE MEAN THEORETICAL INTERCEPT POINT
C     FOR ALL OPTIONS
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(3),ISS,R4
      EXTERNAL F,DR
      DIMENSION S(6),ACL(6),DMY(6),WORK(726),IWORK(5)
      ISS=0
      IGG=0
      IF(IOPT.NE.1)IGG=1
      IF(IOPT.EQ.4)IGG=0
      ICHECK=0
      IF(EL.EQ.0.0)GO TO 2
      ISS=1
      GO TO 20
2     T=TIME+3.0
      CALL ACPOS(T,ACL)
5     AZ=AZMUTH(ACL)
      PXY=SQRT(ACL(1)**2+ACL(2)**2)
      IF(PXY.EQ.0.0)GO TO 10
      EL=ATAN(ACL(3)/PXY)+.02
      GO TO 20
10    EL=ATAN(1.0)*2.
20    VEL=VMUZEL
      S(1)=0.0
      S(2)=0.0
      S(3)=0.0
      VXY=VEL*COS(EL)
      S(4)=VXY*COS(AZ)
      S(5)=VXY*SIN(AZ)
      S(6)=VEL*SIN(EL)
      IFLAG=1
      TINV=TIME
```

```
      TOUT=TIME+10.
      RELERR=RELER
      ABSERR=ABSER
      CALL ODERT(F,3,S,TINV,TOUT,RELERR,ABSERR,IFLAG,WORK,IWORK,DR,
     1REROOT,AEROOT)
      CALL ACPOS(TINV,ACL)
      IF(ISS.EQ.1)GO TO 300
      IF(TINV.LE.TOUT)GO TO 35
      PRINT*,"*****INVALID SOLUTION - INTEGRATION REACHED TOUT W/O MTIP"
      GO TO 300
35    IF(ICHECK.LE.15)GO TO 45
      PRINT*,"*****INVALID SOLUTION - AFTER 15 ITERATIONS A SOLUTION WAS
     1NOT FOUND THAT SATISIFIED THE DELTA RANGE SPECIFIED*****"
      GO TO 300
45    ICHECK=ICHECK+1
      IF(ICHECK.EQ.1)GO TO 5
      IF(CLV(S,ACL).LT.SMIN)GO TO 400
      IF(RANGE(S,ACL).LT..05)GO TO 300
      AZ=AZMUTH(ACL)
      PXY=SQRT(ACL(1)**2+ACL(2)**2)
      IF(PXY.LT..0000001)GO TO 70
      ACR=RANGE(DMY,ACL)
      SR=RANGE(S,DMY)
      ELN=EL+ATAN(2.*(ACL(3)-S(3))/(ACR+SR))
      IF(ABS(ELN-EL).GT..0005)GO TO 60
      DELXY=SQRT((ACL(2)-S(2))**2+(ACL(1)-S(1))**2)
      SSS=((ACR**2+SR**2-DELXY**2)/(2.*ACR*SR))
      IF(SSS.GE.1.)GO TO 60
      DELEL=ACOS(SSS)
      IF(ACR-SR)40,20,50
40    EL=ELN+DELEL
      GO TO 20
50    EL=ELN-DELEL
      GO TO 20
60    EL=ELN
      GO TO 20
```

```
70      EL=ATAN(1.0)*2.
        GO TO 20
300     DO 80 I=1,6
80      TIP(I)=S(I)
        TIP(7)=CLV(S,ACL)
        TIP(8)=TINV
        RETURN
400     WRITE*,"RELATIVE VELOCITY OF SHELL FOR THIS SHOT IS LESS THAN   ",
       1SMIN,"    NO INTERCEPT"
500     RETURN
        END
```

```
      SUBROUTINE DIST(AZ,EL,TIME,VMUZEL)
C
C     SUBROUTINE DIST PROPAGATES THE SHELL TO THE INTERCEPT PLANE
C     AND RECORDS THE POSITION
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      EXTERNAL F,YP
      DIMENSION S(6),ACL(6),WORK(726),IWORK(5),DV(3),R(6),SFR(3)
      IGG=0
      IF(IOPT.NE.1)IGG=1
      CALL ACPOS(TIP(8),ACL)
      IF(IOPT.EQ.3)GO TO 10
      ALPHA=AZMUTH(TIP)-ATAN(1.0)*2.
      GAMMA=ATAN(TIP(6)/SQRT(TIP(4)**2+TIP(5)**2))
      GO TO 25
10    DO 20 I=1,3
20    DV(I)=TIP(I+3)-ACL(I+3)
      ALPHA=AZMUTH(DV)-ATAN(1.0)*2.
      GAMMA=ATAN(DV(3)/SQRT(DV(1)**2+DV(2)**2))
25    BETA=0.0
      CALL TRANS(1)
      VEL=VMUZEL
      S(1)=0.0
      S(2)=0.0
      S(3)=0.0
      VXY=VEL*COS(EL)
      S(4)=VXY*COS(AZ)
      S(5)=VXY*SIN(AZ)
      S(6)=VEL*SIN(EL)
      IFLAG=1
      TINV=TIME
      TOUT=TIME+10.
      RELERR=RELER
      ABSERR=ABSER
```

```
      CALL ODERT(F,6,S,TINV,TOUT,RELERR,ABSERR,IFLAG,WORK,IWORK,YP,
     1REROOT,AEROOT)
      IF(TINV.LE.TOUT)GO TO 30
      PRINT*,"*****INVALID SOLUTION - INTEGRATION REACHED TOUT IN DIST W
     1THOUT A SOLUTION***REDUCE NSAM BY ONE*****"
      RETURN
30    CALL ACPOS(TINV,ACL)
      IF(SIG(4).EQ.0.0)GO TO 40
      TF=TIME-TINV
      DO 50 I=1,3
      SFR(I)=SIG(4)*TF*ACL(I+3)
50    ACL(I)=SFR(I)*R4+ACL(I)
40    ZP=C(1,3)*(S(1)-ACL(1))+C(2,3)*(S(2)-ACL(2))+C(3,3)*(S(3)-ACL(3))
      XP=C(1,1)*(S(1)-ACL(1))+C(2,1)*(S(2)-ACL(2))+C(3,1)*(S(3)-ACL(3))
      SXSQ=SXSQ+XP**2
      SZSQ=SZSQ+ZP**2
      SUMZ=SUMZ+ZP
      SUMX=SUMX+XP
      SXZ=SXZ+XP*ZP
      WRITE(2)XP,ZP
      RETURN
      END
```

```
      FUNCTION YP(T,S,DS)
C
C     FUNCTION YP IS THE USER DEFINED FUNCTION USED BY SUBROUTINE
C     DIST AND CALLED BY ODERT TO DETERMINE WHEN THE SHELL INTERCEPTS
C     THE ENCOUNTER PLANE
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R
      DIMENSION S(6),DS(6),ACL(6),R(6),SFR(3)
      TF=T-(TIP(8)-TOF)
      CALL ACPOS(T,ACL)
      IF(SIG(4).EQ.0.0)GO TO 20
      DO 10 I=1,3
      SFR(I)=SIG(4)*TF*ACL(I+3)
   10 ACL(I)=SFR(I)*R4+ACL(I)
   20 YP=C(1,2)*(S(1)-ACL(1))+C(2,2)*(S(2)-ACL(2))+C(3,2)*(S(3)-ACL(3))
      RETURN
      END
```

```
      FUNCTION DR(T,S,DS)
      DIMENSION S(6),DS(6),ACL(6)
      CALL ACPOS(T,ACL)
      R=RANGE(S,ACL)
      IF(R.LT..0000001)GO TO 10
      DR=((S(1)-ACL(1))*(S(4)-ACL(4))+(S(2)-ACL(2))*(S(5)-ACL(5))+
     1(S(3)-ACL(3))*(S(6)-ACL(6)))/R
      RETURN
10    DR=0.0
      RETURN
      END
```

```
      FUNCTION CLV(S,ACL)
C
C     FUNCTION CLV COMPUTES THE RELATIVE SPEED BETWEEN THE AIRCRAFT
C     AND THE SHELL
C
      COMMON IGG,ACCEL(6),RELER,ABSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION S(6),ACL(6),DV(3)
      DO 10 I=1,3
10    DV(I)=S(I+3)-ACL(I+3)
      CLV=SQRT(DV(1)**2+DV(2)**2+DV(3)**2)
      DC=(DV(1)*ACCEL(4)+DV(2)*ACCEL(5)+DV(3)*ACCEL(6))/CLV
      IF(DC.LT.0.0)RETURN
      CLV=0.0
      RETURN
      END
```

```
      FUNCTION RANGE(S,ACL)
C
C     FUNCTION RANGE COMPUTES THE RANGE BETWEEN THE AIRCRAFT AND
C     THE SHELL
C
      DIMENSION S(6),ACL(6)
      RANGE=SQRT((S(1)-ACL(1))**2+(S(2)-ACL(2))**2+(S(3)-ACL(3))**2)
      RETURN
      END
```

```
      SUBROUTINE GGNOR(ISEED,N,R)
C
C     SUBROUTINE GGNOR COMPUTES NORMAL DEVIATES FROM A UNIFORM
C     RANDOM NUMBER STREAM - USING THE BOX-MJELLER METHOD
C
      DIMENSION R(N)
      PI=ATAN(1.0)*4.
      CALL GGUB(ISEED,N,R)
      DO 10 I=1,N,2
      U1=R(I)
      U2=R(I+1)
      R(I)=SQRT(-2.0*ALOG(U1))*COS(2.0*PI*U2)
10    R(I+1)=SQRT(-2.0*ALOG(U2))*COS(2.0*PI*U1)
      RETURN
      END
```

```
      SUBROUTINE PRINT(NSAM,TIME,IPK,VMUZEL
C
C     SUBROUTINE PRINT IS THE OUTPUT ROUTINE FOR AAASIM
C
     1,STATZ,STATX,KATS,COVAR)
      COMMON IGG,ACCEL(6),RELER,ARSER,REROOT,AEROOT,ALPHA,BETA,GAMMA,
     1SEMIA,SEMIB,SEMIC,HITT,ACLFT(21,8),SXSQ,SZSQ,SUMX,SUMZ,SXZ,SIG(5)
     2,LSEED,TOF,AZM,ELM,IOPT,C(3,3),TIP(8),ISS,R4
      DIMENSION ACL(6),COVAR(3),DMY(6)
      DIMENSION STATZ(3),STATX(3)
      ZMEAN=SUMZ/NSAM
      ZVAR=SZSQ/NSAM-ZMEAN**2
      ZSTD=SQRT(ZVAR)
      XMEAN=SUMX/NSAM
      XVAR=SXSQ/NSAM-XMEAN**2
      XSTD=SQRT(XVAR)
      COV=SXZ/NSAM-XMEAN*ZMEAN
      CALL ACPOS(TIP(8),ACL)
      GO TO(10,20,30,40,50),IOPT
10    PRINT 100
      PRINT 110
60    PRINT 120,TIME,NSAM,SQRT(COVAR(1)),SQRT(COVAR(3)),COVAR(1),
     1COVAR(3),COVAR(2),VMUZEL,SIG(5),(SIG(I),I=1,4)
      PRINT 130,TOF,TIP(8),ACL(1),ACL(2),ACL(3),TIP(1),TIP(2),TIP(3),
     1RANGE(TIP,DMY),RANGE(TIP,ACL),AZM,ELM,TIP(7)
      PRINT 140,ZMEAN,ZSTD,XMEAN,XSTD,COV
      CR=COV/(ZSTD*XSTD)
      PRINT 145,CR
      PRINT 150,KATS,STATZ(1),STATZ(3),STATX(1),STATX(3)
      RETURN
20    PRINT 100
      PRINT 160
      GO TO 60
30    PRINT 100
      PRINT 170
      GO TO 60
```

```
40    PRINT 180
70    PRINT 120,TIME,NSAM,SQRT(COVAR(1)),SQRT(COVAR(3)),COVAR(1),
     1COVAR(3),COVAR(2),VMUZEL,SIG(5),(SIG(I),I=1,4)
      PRINT 130,TOF,TIP(8),ACL(1),ACL(2),ACL(3),TIP(1),TIP(2),TIP(3),
     1RANGE(TIP,DMY),RANGE(TIP,ACL),AZM,ELM,TIP(7)
      CONFL=.95
      CALL BELBIN(NSAM,IPK,.05,PHAT,PLOW,PUP,IER)
      PRINT 200,SEMIA,SEMIB,SEMIC,ALPHA,BETA,GAMMA,IPK,PHAT,PLOW,PUP,CON
     1FL
      RETURN
50    PRINT 190
      GO TO 70
100   FORMAT(1H1,/////,T26,"DISTRIBUTION OF SHELL IMPACT POINTS")
110   FORMAT(T15"OPTION 1  - INTERCEPT PLANE PERPENDICULAR TO LINE OF SI
     1GHT",/,T30"GRAVITY DROP NOT CONSIDERED")
120   FORMAT(/,T14"INPUT PARAMETERS",/,T18"TIME OF FIRE",T53,F13.7T67"SE
     2CS,",/,T18,"SAMPLE SIZE",T59,I7,
     */,T18,"AZIMUTH AIMING ANGLE STD.DEV.",T53,F13.7,T67,"RADIANS",
     */,T18,"ELEVATION AIMING ANGLE STD.DEV.",T53,F13.7,T67,"RADIANS",
     3          /,T18"AZIMUTH AIMING ANGLE VARIANCE",T53,F13.7/,T1
     48"ELEVATION AIMING ANGLE VARIANCE",T53,F13.7/,T18"AIMING ANGLE COV
     5ARIANCE",T53,F13.7/,T18"MUZZLE VELOCITY",T53,F13.7T69"M/SEC",/,T18
     6"MUZZLE VELOCITY STD.DEV.",T53,F13.7T69"M/SEC",/,T18"BALLISTIC DIS
     7PERSION STD.DEV.",T53,F13.7T67"RADIANS",/,T18"ATMOSPHERIC DISTURBA
     8NCE STD.DEV.",T53,F13.7T67"RADIANS",/,T18"GUN JITTER STD.DEV.",T54
     9,F12.7T67"RADIANS",/,T18"FLIGHT ROUGHNESS COEFFICIENT",T53F13.7)
130   FORMAT(/,T14"MEAN THEORETICAL IMPACT POINT (MTIP)",/,T18"TIME OF F
     1LIGHT OF SHELL",T53,F13.7T69"SECS.",/,T18"TIME OF MEAN INTERCEPT",
     2T53,F13.7T69"SECS.",/,T18"POSITION OF AIRCRAFT AT MTIP",/,T22"X-CO
     3ORDINATE",T53,F13.7/,T22"Y-COORDINATE",T53,F13.7/,T22"Z-COORDINATE
     4",T53,F13.7/,T18"POSITION OF SHELL AT MTIP                 ",/,T2
     52"X-COORDINATE",T53,F13.7/,T22"Y-COORDINATE",T53,F13.7/,T22"Z-COOR
     6DINATE",T53,F13.7/,T18"RANGE OF SHELL",T53,F13.7T68"METERS",/,T18"
     7SHELL/AIRCRAFT DIST ERROR AT MTIP      ",T57,F9.6T68"METERS",/,T18"M
     8TIP AZIMUTH ANGLE",T53,F13.7T67"RADIANS",/,T18"MTIP ELEVATION ANGL
     9E",T53,F13.7T67"RADIANS",/,T18"RELATIVE SPEED AT MTIP    ",T53F13.7
```

```
     *,T69"M/SEC")
140   FORMAT(/,T14"DISTRIBUTION PARAMETERS",/,T18"VERTICAL MEAN",T54F12.
     16,T68"METERS",/,T18"VERTICAL STD.DEV.",T53F13.7T68"METERS",/,T18"H
     2ORIZONAL MEAN",T53,F13.7T68"METERS",/,T18"HORIZONTAL STD.DEV.",T5
     34F12.7,T68"METERS",/,T18"VERT/HORZ POSITION COVARIANCE",T53F13.7)
145   FORMAT(T18,"CORRELATION COEFFICIENT",T53,F13.7)
150   FORMAT(/,T14"GOODNESS OF FIT TO NORMAL DISTRIBUTION",/,T18"NUMBER
     1OF CATAGORY CELLS",T59,I7/,T18"VERTICAL CHI-SQUARE STATISTIC",T54,
     2F12.7/,T18"VERTICAL SIGNIFICANCE LEVEL",T53,F13.7/,T18"HORIZONTAL
     3CHI-SQUARE STATISTIC",T53,F13.7/,T18"HORIZONTAL SIGNIFICANCE LEVEL
     4",T53,F13.7)
160   FORMAT(T14,"OPTION 2  - INTERCEPT PLANE PERPENDICULAR TO SHELL VEL
     1OCITY",/,T32"GRAVITY DROP CONSIDERED")
170   FORMAT(T14,"OPTION 3 INTERCEPT PLANE PERPENDICULAR TO RELATIVE VEL
     1OCITY",/,T32"GRAVITY DROP CONSIDERED")
190   FORMAT(1H1,/////,       T14,"OPTION 5 - PROBABILITY OF HIT ON ELLIP
     1SOIDAL SHAPED AIRCRAFT",/,T32"GRAVITY DROP CONSIDERED")
180   FORMAT(1H1,/////,       T14"OPTION 4 - PROBABILITY OF HIT ON ELLIPS
     1OIDAL SHAPED AIRCRAFT",/,T30"GRAVITY DROP NOT CONSIDERED")
200   FORMAT(/,T14"SIZE OF ELLIPSOIDAL AIRCRAFT",/,T18"X SEMI-AXIS",T54,
     1F12.7T68"METERS",/,T18"Y SEMI-AXIS",T53,F13.7T68"METERS",/,T18"Z S
     2EMI-AXIS",T53,F13.7T68"METERS",/,T14"AIRCRAFT ATTITUDE RELATIVE TO
     3 GUN",/,T18"HEADING",T53,F13.7T67"RADIANS",/,T18"PITCH",T53,F13.7T
     467"RADIANS",/,T18"ROLL",T53,F13.7T67"RADIANS",/,T14"NUMBER OF HITS
     5",T59,I7,/,T14,"PROBABILITY OF HIT",T53,F13.7,/,T14,"CONF. INTERVA
     6L OF PROBABILITY OF HIT ",/,T18,"LOWER BOUND",T53,F13.7,/,T18"UPPE
     7R BOUND",T53,F13.7,/,T14,"CONFIDENCE LEVEL",T53,F13.7)
      END
```

```
      SUBROUTINE NTEST(KATS,STATZ,STATX)
      DIMENSION X(1000),Z(1000),COVAR(3),DUM(3),CA(2,2),B(3),WKAREA(6),
     1C(3),OBSC(20),CSOBS(20),STAT(3),ARRAY(1000),STATZ(3),STATX(3)
C
C     NORMALITY TEST,MONTE CARLO VERSION,AAA SIMULATION
C
10    REWIND 2
20    READ (2)TIME,NSAM
      DO 40 I=1,NSAM
40    READ(2)X(I),Z(I)
      READ(2)SXSQ,SZSQ,SUMX,SUMZ,SXZ
      ZMEAN=SUMZ/NSAM
      COVAR(3)=SZSQ/NSAM-ZMEAN**2
      XMEAN=SUMX/NSAM
      COVAR(1)=SXSQ/NSAM-XMEAN**2
      COVAR(2)=SXZ/NSAM-ZMEAN*XMEAN
C
C     CHOLEVSKY DECOMPOSITION
C
      CALL LUDECP(COVAR,DUM,2,D1,D2,IER)
C
C     EXPAND TO FULL STORAGE MODE
C
      CA(1,1)=1.0/DUM(1)
      CA(2,1)=DUM(2)
      CA(2,2)=1.0/DUM(3)
      CA(1,2)=0.0
C
C     INVERT FACTOR  MATRIX
C
      D1=-1.0
      CALL LINV3F(CA,B,1,2,2,D1,D2,WKAREA,IER)
C
C     TRANSFORM ARRAYS TO GET INDEPENDENCE
C
      DO 50 J=1,NSAM
```

```
      B(1)=X(J)-XMEAN
      B(2)=Z(J)-ZMEAN
      CALL VMULFF(CA,B,2,2,1,2,3,C,3,IER)
      X(J)=C(1)
      Z(J)=C(2)
50    CONTINUE
C
C     TEST FOR NORMALITY BY COMPONENT
C
      DO 200 JJ=1,NSAM
200   ARRAY(JJ)=X(JJ)
      DO 80 K=1,2
      DO 70 L=1,KATS
      OBSC(L)=0.0
      CSOBS(L)=0.0
70    CONTINUE
      STAT(3)=1.0
      CALL GTNOR(ARRAY,NSAM,KATS,STAT,OBSC,CSOBS,IER)
      DO 100 II=1,NSAM
100   ARRAY(II)=Z(II)
      IF(K.EQ.2)GO TO 104
      DO 105 LL=1,3
105   STATX(LL)=STAT(LL)
      GO TO 80
104   DO 107 KK=1,3
107   STATZ(KK)=STAT(KK)
80    CONTINUE
      RETURN
      END
```

Vita

Errol Wilkins was born on 1 February 1942 in Louisville, Kentucky and graduated from duPont Manual High School in Louisville in 1960. He attended the University of Louisville Speed Scientific School where he earned his Bachelors Degree in Mechanical Engineering and a commission in the Air Force through ROTC in 1965.

Errol graduated from Undergraduate Navigator Training, as a distinguished graduate, in 1967 and moved to Yokota, Japan for a tour with the 56th Weather Reconnaissance Squadron, flying WC-135s. In September 1970 he began his one year tour in Vietnam with the 361st Tactical Electronics Warfare Squadron, flying EC-47s. His next duty assignment was back at Mather AFB as an instructor in the Undergraduate Navigator Training School, where he participated in the operational test and evaluation of the new T-43. In August of 1975 he moved to Langley AFB, Va. with the 6th Airborne Command and Control Squadron and entered the Air Force Institute of Technology in August 1977 as a student in the new Strategic and Tactical Science program.

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Sensitivity Analysis
Antiaircraft Artillery Model

P sub k

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study was conducted to investigate the sensitivity of aircraft probability of kill (P_k) estimates to the parameters of a supplied antiaircraft artillery aiming angle distribution. A model was developed using basic physical relationships for the shell propagation and Monte Carlo sampling techniques to incorporate the random errors into the model. A shaped target was used and all random error sources were assumed to be bivariate normally distributed with zero means. ~~(over)~~

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Simulation runs were made holding all variables constant except for the aiming angle variances and response curves were constructed by plotting P_k versus aiming angle variance. Tests were also conducted to test the assumption that the shell distribution on the encounter plane was bivariate normal and that the effects of gravity drop on the shell could be neglected. The results from the study model were compared with simular results from model P001.

The sensitivity of the P_k estimates was found to be a function of the magnitude of the P_k, the aiming angle variance, the sum of the ballistic error variances, and the aircraft vulnerable area. Response curve trends indicated slightly more sensitivity from the study model than from P001, however, it could not be concluded with certainty.